Master Thesis

Solving the Profit Sharing Problem through Negotiations with Information Confirmation Processes

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Abstract

These days, the profit sharing problem is occurring in several fields. If the field is joint-stock cooperation, we can divide the profit with the ratio of its stock holders' holdings, but if there is not hierarchical structure, and we cannot determine imputation with top-down, we cannot solve the profit sharing problem simply. For example, the profit sharing problem in the composite web service which is made by several atomic services, the problem how to divide the profit produced by the composite web service is difficult. In these kinds of problems we can apply theory of cooperative game. Especially, we use n player cooperative game, which is for how to make the coalition with the members and how to divide the profit produced by the coalition. For example, *Shapley* value proposed in cooperative game theory enables fairly division in sense of the value satisfies several axioms. But to calculate imputation with cooperative game theory like *Shapley* value, we need to know all characteristic function values which are the values corresponding to all subset coalition. So, it is difficult to apply to real cases and the value is only applied to particular problems. In this research, I am going to realize fair profit sharing with incomplete information in characteristic function values. Especially, the problem with incomplete information in characteristic function values but we can know characteristic function values with the cost in some measure. This is a way of thinking that we can know the value of the coalition through the service's user's reputation against composite web service. In such situation we can know characteristic function values with the cost, there are some problems as follows.

1. There are few discussion about the situation we can know characteristic function values with the cost, so we cannot deal with the problem on accomplished structure. We need to formalize the problem settings.
2. Which protocol is appropriate for fair imputation in the situation. Each player act to maximize his or her profits, in this situation we need to think what is the protocol which reflects players' contribution most.

3. When we do not know characteristic function values, to calculate imputation such as *Shapley* value, how to treat unknown characteristic function values is unobvious.

In this research, I consider how to treat unknown characteristic function values and formalize problem setting including players' knowledge and characteristic function values. I consider what will be the imputation when each player goes through with his or her strategy.

Three points of the contributions of this paper is below.

1. Problem setting: I proposed problem setting and formalization to deal with the situation all characteristic function values are not known but we can know with the cost. In this problem setting we will be able to treat the problem with cooperative game.
2. Proposal of the protocol: I proposed the protocol which enables us to realize fairly profit sharing with paying cost.
3. Treatment of unknown characteristic function values: We calculate *Shapley* value with unknown characteristic function values, under assumption of unknown characteristic function values as zero.

In these contributions, I show we can make profit sharing under the situation which we do not know all characteristic function values in some condition.

情報確認過程を含む交渉による収益配分問題の解決

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内容梗概

現在、さまざまな分野において利益の分配に関する問題が発生している。株式会社などの場合、利益を株数の比に応じて分配すれば事足りるが、利益を分配する当事者間に権力構造などがなく、トップダウンで決定されない場合には単純には解決できない。たとえば複合 web サービスのようにいくつかの原子サービスが組み合わされたサービスにおいて生み出した利益を原子サービスの提供者間で分配する問題などである。こういった問題の解決には協力ゲーム理論を用いることができる。特に n 人協力ゲームというプレイヤー間でどのように提携関係を結ぶか、その結果得られた利得をプレイヤー間でどのように分け合うか、あるいは分け合うべきかということを対象にしている枠組みを使う。例えば協力ゲーム理論の中で提案されている Shapley 値という配分の方法はいくつかの公理を満たすという点で納得性の高い配分を可能にしている。しかしこうした協力ゲーム理論における配分の計算のためには特性関数という、配分を求めたい提携の部分集合である全ての提携に対応した値を知っている必要があり、そのために現実の問題には応用しにくく、特定の問題にのみ使われている。本研究では特性関数値が完全にはわからない場合においても参加者間で納得のいく利益分配を実現することを目的とする。特に問題を、特性関数値が全部は分からないが、費用をかけることによって特性関数を知ることができる場合に限定し、協力ゲーム理論の知見を用いて利益の分配を可能にする方法を考える。複合 web サービスにおいては、各サービスの組み合わせからなる複合サービスはコストをかけることによってユーザによって評価されうるという考え方である。問題設定を定式化し、利益を分配するためのプロトコルを提案した上で、参加者がどう行動するかを検討する。費用をかけて特性関数を知ることができる場合の利益分配においては、以下のような問題が発生する。

1) 特性関数値が分からない問題のうち、費用をかけることにより特性関数を獲得することができる場合はこれまで十分に議論されていない。そのため協力ゲーム理論で扱えるよう問題を定式化する必要がある。

2) 各プレイヤーの利益が適切に分配されるためのプロトコルはどういったものになるか。各プレイヤーは自分の利益を最大化しようと行動する。このとき

に各プレイヤーの意志を反映して、適切な利益分配を可能にするプロトコルを考える必要がある。

3) 特性関数値が未知である提携をどう扱うべきか。特性関数値が分からない場合においてShapley値などの配分を計算するためには、未知の特性関数をどう扱うか検討する必要がある。

本研究では、未知の特性関数値をどう扱うか検討し、上で各参加者の知識と特性関数値を規定した問題設定を定式化し、どういった手順で未知の特性関数値に対して費用をかけて調べていくかをプロトコルとしてまとめる。このプロトコル上での各参加者の戦略を実行した場合に配分がどうなるのかを検討する。

本研究における貢献は次の3点である。

1) 問題設定。初期状態ではすべての特性関数値は未知であるが、費用をかけることにより特性関数値を獲得することができる場合をゲームとして検討し、協力ゲーム理論で扱えるよう問題を定式化した。

2) プロトコルの提案。プレイヤー間で利益を分配し、費用を負担しながら納得性の高い配分を実現できるプロトコルを提案した。

3) 特性関数値が未知のもの場合に0または優加法性を満たす最小の大きさであるとして計算する。これは特性関数が分からないということはその提携は価値を生み出していないため、知られていないという解釈のもとである。これにより、特性関数値に欠損があっても一定のゲームの解を計算することが可能になる。

以上の貢献により、一定の条件のもとではあるが、特性関数値が全ては分からないような利益分配の問題においても協力ゲーム理論を用いた納得性の高い利益分配が可能になることを示した。

Solving the Profit Sharing Problem through Negotiations with Information Confirmation Processes

Contents

Chapter 1 Introduction

In this paper I treat a problem of profit sharing. This is the problem about how to divide the profit produced by a number of participants with the participants. The composite web service is the example of profit sharing. This is a service made by a number of atomic service.These days, technology's advance such as SOAP or REST made ease to make composite web service. Splitting off of the service interface and service implementation, once implemented service could be applied various form. But there are not so many useful open atomic services. One of the reason is difficulty of sharing profit made by composite service.So, the composite web service has to be free use only and there is little merit for provider of atomic service. This is because the method for profit sharing is counted on to be developed. For example, the Language grid which is composed by many language resources [1] is one of the composite web service. There are a number of translation services between Japanese and English, and dictionary for various areas. In combination of this atomic service, we can use variety of service. Now, all service are provided for free because of the difficulty of profit sharing. But if the composite service can earn money by advertisement or charge for users, we can share the profit with the provider of the services and needs the method for sharing. If we are able to share the profit like this situation, atomic service provider can earn with his or her atomic service not only consistent service. And the merit for provider will be increase and an incentive to provide atomic service will be strong. Development of this method is expected to make a market active both atomic service and composite service.

We think how to solve profit sharing problem from several study. This problem is studied long time and several useful methods are proposed in game theory and micro economics. There are three typical way of thinking. Let's explain easily. First, contract theory, this treat phenomenon relate to dealing of goods and service, and goal to solve problem in contract such as asymmetric information. So, it is difficult to apply imputation after produced profit, directly. Second is bargaining game, this is the way how participants make their decision on bargaining on mutual dependence. In bargaining game there are many studies how to reach equilibrium or what is the equilibrium. Third cooperative game theory, there are two main way, how to make coalition when participants make some value and how to divide the produced profit by some coalition. Each method has each appropriate situation, in our problem, cooperative game theory is the most appropriate because it can treat ex-post profit sharing. If we can calculate solution of cooperative game, it will be the optimal imputation. But to apply cooperative game theory it happens some problems. It is because we can calculate the solution of the game in specific case. For example, the *Shapley* value, which I will explain next chapter, is the one of the most appropriate value in meaning of the value satisfies several axiom. But to calculate the value, we need all characteristic function values, which is the value that corresponds to all subset coalition of the grand coalition. There are two to the number of players power of characteristic function values and there are little case which we can know all characteristic function values. If the number of player increases, amount of calculation increase explosively. So most applications are for special case we can calculate characteristic function values with some rules. In some case there are missing values, the method to calculate approximately is proposed but most of it is simple game which has only one or zero as characteristic function values. The simple game is mainly used for voting game to calculate voting power of party. To calculate profit sharing problem with approximation method the error increase too big to use real problem. Second problem is computational time and amount of calculation. Calculation of *Shapley* value is known as NP sharp problem and if we know all characteristic function values, we need large amount of calculation when participants increase. In this research, I do not need real-time calculation, so I do not consider this problem now. If we do not know characteristic function values without grand coalition, we have to divide the profit equally because of there are no information about participants' contribution [2]. But this method is not fair when there is difference between participants' contribution for whole profit. So, it is need to reflects participants contribution to imputation. There is little discussion about how to get information of characteristic function values. This research's goal is below three points.

1. How to deal with incomplete information in profit sharing. Almost always case we do not know all characteristic function values. In such situation I consider to apply cooperative game theory's knowledge to share the profit.
2. Treatment of unknown characteristic function values: There are a little discussion about how to complement unknown characteristic function values to calculate *Shapley* value.
3. What protocol and strategy will be suitable in such situation.

In this paper, I will explain background and position of this research in second chapter, problem setting in third chapter, protocol and strategy in fourth chapter and evaluation fifth chapter.

Chapter 2 Background

In this chapter, I will explain framework of traditional problem setting of profit sharing. And then I will explain relational study and problem I will treat in this research.

2.1 Traditional Method for Profit Sharing

We think how to solve profit sharing problem from several study. This problem is studied long time and several useful method are proposed in game theory and micro economics. There are three typical way of thinking [3],[4]. First, contract theory, this treat phenomenon relate to dealing of goods and service, and goal to solve problem in contract such as asymmetric information. So, it is difficult to apply imputation after produced profit, directly. Second is bargaining game, this is the way how participants make their decision on bargaining on mutual dependence. In bargaining game there are many studies how to reach equilibrium or what is the equilibrium. Third cooperative game theory, there are two main way, how to make coalition when participants make some value and how to divide the produced profit by some coalition [5], [6].

Each method have each appropriate situation, in our problem cooperative game theory is the most appropriate because of it can treat ex-post profit sharing.

If we can calculate solution of cooperative game, it will be the optimal imputation. But to apply cooperative game theory there happen some problems. It is because we can calculate the solution of the game in specific case [7], [8]. For example, the *Shapley* value, which I will explain next chapter, is the one of the most appropriate value in meaning of the value satisfies several axiom. But to calculate the value, we need all characteristic function values, which is the value that corresponds to all subset coalition of the grand coalition.

2.2 Cooperative Game

I will explain *Shapley* value one of the solution of the game, which represents expected value before participants join the game. *Shapley* value is the only value which satisfies five axiom, individual fairness, efficiency, symmetry, additively, and null player. This means the value is mathematical fair value.

Shapley value is one of the solution of the value in transferable utility game. And which represent participants' expected profit before player joins the game. Shapley value proposed by Shapley 1953 [9]. In cooperative game, we represent participants of the sharing as player of the game. A set of players as coalition, particularly subset of player set N as coalition. The value will be produced by the coalition is characteristic function value of the coalition and represented v. For example combination of player A and B represents coalition $\{A, B\}$ and the value of the coalition $\{A, B\}$ is $v(\{A, B\})$. For example, a coalition of transfer engine A and expert term dictionary B cooperate a coalition $\{A, B\})$, this characteristic function value is $v(\{A, B\})$. In game which N player join, there is 2^n numbers of coalition and its characteristic function value includes grand coalition and empty set.

Now, I will explain the property *super-additive* which is assumed in cooperative game. If the game satisfies *super-additive* all coalitions satisfy below condition.

$$v(S \cup T) \geq v(S) + v(T) \forall S, T \subseteq N(S \cap T = \emptyset)$$

A marginal contribution of the player means the difference between a characteristic function values of the coalition include the player and the characteristic function value of the coalition without the player. That means the additional profit brought by the player.

I will explain strict definition of the marginal contribution below. *Shapley* value is expected value of player's marginal contribution. That is *Shapley* value represent contribution of the player to whole profit.

In profit sharing in composite web service, characteristic function values represent user's reputation and value for the service of the coalition. Dividing with the ratio of Shapley values mean dividing with the ratio of each service's contribution.

Shapley value of player i is defined as below.

$$\phi_i(v) = \frac{1}{n!} \sum_{\Pi \in \Pi} \{v(S_{\Pi,i} \cap \{i\}) - v(S_{\Pi,i})\}$$

In this Π is the set of permutation 1 to n, $v(S_{\Pi,i})$ means the set of the players who line before player i. This definition can be interpreted as below. Think the situation that a number of n players gather one by one. Now, players lined in a order f $\Pi \in \Pi$. Then player i join and characteristic function values be increase $v(S_{\Pi,i} \cap \{i\}) - v(S_{\Pi,i})$.

This value is defined as player i's marginal contribution. in order Π. A number of n player gather in random order, then one order will come in probability of $\frac{1}{n}$. Above $\phi_i(v)$ shows expectation of player i's marginal contribution.

I will show way to calculate *Shapley* value when there is three players below.

1. set up all order of player A, B, C
2. Order ABC shows player gather in order of $A \rightarrow B \rightarrow C$ So, the marginal contribution of A will be $v(\{A\})$, B will be $v(\{A, B\})$ - $v(\{A\})$, C will be calculated $v(\{A, B, C\})$ - $v(\{A, B\})$,
3. Set out all marginal contribution of each player with all orders.
4. Make average marginal contribution of each player's order as *Shapley* value.

We will show how to calculate *Shapley* value in blunt terms

Let's make characteristic function values as table 1

Table 1: Example : characteristic function values

coalition	characteristic function value
{A}	8
{B}	5
{C}	3
{A,B}	15
{B,C}	11
{C,A}	14
{A,B,C}	20

In this case, each player's *Shapley* value is calculated as 2

Table 2: Example of *Shapley* value

Player	*Shapley*value
A	55/6
B	37/6
C	14/3

2.3 Relational Study

I will explain relational study of cooperative game theory

1. MC net

   MC net means Marginal Contribution net this represent characteristic function representation compact, and ease calculation. This makes amount of calculation small and contributes to easy calculation of the Shapley value.This do not represents characteristic function values directly but increase of the characteristic function value. That is marginal contribution. This method has merit for example, we can calculate *Shapley* value with linear computational time. But this is same we have to know all characteristic function values to calculate *Shapley* value.

2. Bidding Mechanism It enables us treat profit sharing in cooperative game as non-cooperative game. This means player act rationally for his or her profit. Bidding Mechanism use bidding the protocol of auction and decide imputation. This mechanism is showed this game's sub-game perfect equilibrium reach each players' *Shapley* value [10]. This mechanism decide proposer by bidding and the highest bidder becomes proposer. The proposer pays his or her bid to other players. And proposer proposes players' imputation, if the imputation was accepted by other players, the game ends. If the propose rejected, the proposer be removed and the game goes on. This mechanism is expected to apply cost allocation problem with auction and resource allocation with multi-agent simulation.

3. Calculation Approximately There are many studies about calculation approximately, but almost all of this is about simple game. Simple game is applied voting game and studied well. This game's characteristic function

value has only one or zero. It means votes success or fail. That is the coalition's vote success or fail. In this game *Shapley* value is used as voting power of the party. In this simple game there are several study of the way how to decrease amount of calculation or solution for missing value [11, 12].

2.4 Requirements

I will show the demanded condition or properties for protocol of profit sharing problem.

- pareto efficiency
- Null player
- symmetry
- additively
- Calculation easiness
- Strategy proofness
- decidability

Shapley value has first four properties. We will discuss the solution I propose satisfies what properties and not. This represents solution's performance. Pareto efficiency: sum of the imputation is same to grand coalition. This means without decrease of some player's profit we cannot increase other player's profit. Null player is the player who did no contribution on any coalition. This property means null player's imputation become zero. Symmetry: two players who contributed same to all coalition, same imputation. That is indistinguishable player should be same imputation. Additively means think player A and player B as one player, then his imputation should be same as player A's plus B's. Calculation capability is one of the metrics for amount of calculation and computational time. In this research, I do not treat this metrics. Strategy proofness: Honesty is the best way for the player whatever other player say. This is the properties for mechanism. This is important property for mechanism because player can trust the protocol. Decidability means we can reach same result from same initial state. If we can guarantee this property the protocol's stability will grow. But in this research I assume incomplete information for players each other and some random, it is difficult to guarantee this property.

2.5 Purpose and Solution

As we observed, it is desirable to use *Shapley* value in profit sharing problem if the problem has complete information. But in real problem it makes even less sense that we know all characteristic function values. Then, we want to realize profit sharing. But if all information are unknown, it is unmanageable. So I will limit problem as we can treat. In the situation, we cannot know characteristic function values, there is some case we can know the missing characteristic function values with cost. For example, in Composite web service, we can know some coalition's characteristic function value with user reputation. In this research I consider how the profit be shared in such situation. In such situation it is not obvious how to reach fair imputation. Especially I aim to reach good imputation with less paying the cost.

In particular I think the situation that information of characteristic function values is asymmetry. In composite web service, the characteristic function values are known to only the service coalition's constituent atomic service provider. This is from the assumption of constituent member know his or her contribution to the coalition. In this case, each player acts to maximize his or her gain. But player do not know all characteristic function values and cannot calculate true *Shapley* value. And the characteristic function value shared to all players is only grand coalition, in initial stage. If we calculate from shared knowledge, it may be dividing equally under assumption unknown characteristic function values as zero. On another front, if profit has divided equally, the player contributed a lot will have excess or dissatisfaction. And the player is going to make known his or her contribution by making known his or her characteristic function value. Then player pay some cost for making known to other players. This means user or some agent evaluate the service with cost. This is information confirmation process. In this paper, I consider the mechanism on above condition and what problem setting is suitable to treat as cooperative game and how imputation reaches fair level.

I think below two purposes.

1. In the environment of composite web service, protocol should be smooth and available on various case easily.

2. Guarantee mechanism and solution has several useful properties.

2.6 Problem in Application on Composite Web Service

I will explain relation between problem setting and real problem, especially composite web service. Think when some composite web service produces a profit. Then the problem how to divide the profit will happen. If there is some platform service or hub service which have a power, profit sharing goes on smoothly with preliminary contracted process. But if all service providers are equal basis, it is difficult. This is because composite web service could be made by ordinary people and contract in each case cost a lot. And the profit the service will make is difficult to forecast. If the service produce big amount of profit, then providers have to think how to divide the profit. In such case, I think profit sharing by cooperative game theory. In composite web service, the value of the coalition that is characteristic function values is evaluated by users. It means users use the service and decide the evaluation of the service. In some service we can decide the evaluation by page view, revenue per user or rate of advertising but many services it is difficult to decide. It is safe to assume that to make problem easy, we think user's evaluation like questionnaire, interview or experiment as a user revaluation. But users are interesting in using service not evaluating each service. If we force users to evaluate, users will not use the service in meaning of avoidance load. Then we think pay the reward for user's evaluation on the service. This means pay appropriate reward for user's evaluation burden. One of the methods for profit sharing is egalitarian rule, that is dividing equally. Dividing equally means calculating *Shapley* value with grand coalition's characteristic function value and think other characteristic function values as zero. It do not need other characteristic function values and user evaluation.

But this method results same imputation of core competency, low fungibility service and simple, fungible service. And it will result unfair imputation. If atomic web service providers do not accept the dividing equally, the provider will withdraw or deny joining the grand coalition. On another front if we think the situation we make users evaluate all service. In this case we know all characteristic function values and can calculate *Shapley* value or *core* and other solution of the game proposed in cooperative game theory. In this, we can propose imputation. But, this we load user a lot, and users may escape. That is, dividing equally by making users evaluate nothing or evaluate all characteristic function values. This is because I think it is needed to propose method for fair profit sharing with knowing a portion of characteristic function values. In this case the problem is how to choice characteristic function values to evaluate. In calculating *Shapley* value, when $v(A) = 1$ and $v(B) = 10$ it is less influence for *Shapley* value to skip out $v(A) = 1$ that is it means not evaluate $v(A) = 1$. But *Shapley* value satisfy coalition monotonicity and *super-additive* , all atomic service provider will hope evaluate all characteristic function values which coalition includes the player. So, I think charge provider to evaluate cost for users, and hold down evaluating less influential characteristic function values. That is why information confirmation process is needed.

2.7 Unknown Characteristic Function Values

In this chapter I will explain unknown characteristic function values. In my proposal problem settings, basically I assume unknown characteristic function values as zero. But in characteristic function game there is a assumption of *super-additive* . So I assume unknown characteristic function values of coalition s as characteristic function value of subset coalition s' which has the biggest characteristic function value. If I do not know a coalition's characteristic function value then I can consider the coalition has no value. Let's explain it with simple example. If I think, some composite web service of machine translation service which treat several language resources. There are atomic service follows.
A : Jserver
B : GoogleTranslation
C : Multilingual dictionary for Specific Terms
And characteristic function value is set as below. $v(\{A, B, C\}) = 150$ (grand coalition)
$v(\{A, C\}) = 60$
$v(\{B, C\}) = 70$

$(v(\{A,B\}) = 65)$ (Essentially, this composite service do not work, but set $v(A) + v(B)$ to satisfy *super-additive* .) $v(\{A\}) = 30$
$v(\{B\}) = 35$
$v(\{C\}) =?$ (I set zero because the service c not works itself)Now it is problem how to treat $v(\{c\})$, we can consider c contributes something. But we can think the contribution is woven into the other characteristic function such as $v(\{A,C\}) - v(\{A\})$. Then if we add something to $v(\{c\})$, then it will have an arbitrariness.

Then, I think to set unknown characteristic function values stochastically from the assumption of *super-additive* . For example in the above case, I think $v(\{C\})$ as the value $0 \sim min(v(\{A,C\}) - v(\{A\}), v(\{B,C\}) - v(\{B\}))$. Now, if we can calculate *Shapley* value with the characteristic function values represented as stochastical form. But this is very difficult even I can know the distribution of the value. It is because the value may be represented as recursive form like next example.

Example)
$v(\{A,B,C\}) = 100$
$v(\{A,C\}) = 70$
$v(\{B,C\}) : unknown \ (v(\{B\}) + v(\{C\}) \leq v(\{B,C\}) \leq 100 - v(\{A\})) \Rightarrow 50 \leq v(\{B,C\}) \leq 100 - v(\{A\})$
$v(\{A,B\}) : unknown \ (v(\{A\}) + 30 \leq v(\{A,B\}) \leq 100 - v(\{C\})) \Rightarrow v(\{A\}) + 30 \leq v(\{A,B\}) \leq 80$
$v(\{A\}) : unknown \ (0 \leq v(\{A\}) \leq min(v(\{A,C\}) - v(\{C\}), v(\{A,B\}) - v(\{B\})) \Rightarrow 0 \leq v(\{A\}) \leq min(50, v(\{A\}))$
$v(\{B\}) = 30$
$v(\{C\}) = 20$

Whichever method I take it need real data to consider, now easiness to treat and above explanation, I consider unknown characteristic function values as zero.

2.8 Consideration for Problem Setting

It is nott possible to use a solution of cooperative game theory directly for a solution of a profit sharing problem of the composite web service as I have checked to here. When comparing, a solution of a cooperative game theories *Shapley* value. The condition to find the characteristic function values completely to apply to a real problem is hard. But it is difficult as the profit sharing in the case when we do not know the characteristic function values as I explained preceding section.

I restrict a problem because it is difficult to handle the unknown case when the characteristic function value is not perfection. In the same way, the problem that I do not know the characteristic function value, considers a case possible to know the characteristic function value. It is when I think an example of composite web service, the user evaluates the value of the cooperation, and it is possible to know the characteristic function value. I think take up the case it is possible to know the unknown characteristic function value by some means. It is when it is possible to know the characteristic function value by paying for the cost.

When the characteristic function value can be evaluated here, I think what kind of setting is needed. Supposes every player has no background knowledge information. Even if players evaluate the characteristic function value at this time, the player do not know which of the characteristic function value he or she should estimate. Further, it'll be proper to think a player has knowledge to a little characteristic function value from the supposition that a player assumes contribution in his cooperation. We assume that there is expectation of the characteristic function value certain degree of to the coalition with which one participates join. In other words, the characteristic function value of the cooperation with which one participates is possible to think the problem saying to know.

Now, in such case, by what kind of procedure can I make them to achieve the profit sharing? I need a method that the cost is not consumed as much as possible, and difference with the player's ideal imputation small.

In such case what happened to the profit sharing and after putting supposition in the knowledge of each player, considered whether it is not possible to handle it. A player makes the characteristic function value well-known by knowing only the related characteristic function value and paying the fixed cost to deal with the problem that it couldn't be handled by a structure of theory of a conventional cooperation game.

I formulated the framework to calculate Shapley value from the characteristic function value all players known and common.

In this structure I expressed what kind of behavior each player would get as a protocol, and considered with what kind of strategy he or she behaved on it. In the result, I showed that this protocol reach fair profit sharing in such problem setting.

2.9 Contribution

In this research, I focus on situation that all characteristic function values are not shared, and aim to apply cooperative game theory's knowledge. And I will propose mechanism which enables fair imputation in condition that unknown characteristic function could be evaluated by paying cost.

Table 3: Knowledge when there are three players

player A	player B	player C
v({A})=8	v({B})=5	v({C})=3
v({A,B})=15	v({A,B})=15	v({A,C})=14
v({A,C})=14	v({B,C})=11	v({B,C})=11
v({A,B,C})=20	v({A,B,C})=20	v({A,B,C})=20

Chapter 3 Problem Setting

In this chapter I will show problem setting of profit sharing when there are unknown characteristic function values, especially the difference from traditional cooperative game.

3.1 Knowledge of the Players

Under assumption of player can predict his or her contribution to his or her coalition, player knows characteristic function values of the coalition his or her join. It means a player know a half of the characteristic function values, a number of 2^{n-1}.

Now I marshal the knowledge players can use. There are the characteristic function values which coalition he or she joins and shared knowledge the fixed cost clearly, and temporal imputation, how much other player paid, and from the assumption of *super-additive* , the player can know upper bound of unknown characteristic function values.

Table 3 is initial knowledge of each players Eight coalition includes null coalition $\{\emptyset\}$, each player knows four characteristic function values.

I assume players' knowledge is asymmetry and player knows the characteristic function values which coalition he or she joins. But I do not think players share his or her knowledge by other players. This is why there is no guarantee players tell true characteristic function values, that is to increase his or her gain player has incentive to tell bigger value. In real problem, player may not know the characteristic function values which coalition concretely. In the situation it is difficult to share fuzzy value. Player calculate temporary *Shapley* value from his or her prediction of the characteristic function values. And if his or her profit would increase, he or she pay the cost to evaluate the characteristic function value. In this research, I simplify the problem to treat the problem with cooperative game theory and I do not suppose player exchange his or her knowledge.

3.2 Evaluator

At this chapter I think about the evaluator who decides about the value of the coalition. This is the function of the value of coalition or the value coalition would invent in this problem setting. This is clogged and means characteristic function value in the coalition. It is charged with the role of making all the members make unknown characteristic function value which is not shared by all the members known. That I think in composite web service, an evaluator, it may be regarded as the user who estimates the value of coalition we'd like to purchase. When it is an enterprise, it can be also the market who decides about its value. Additionally it may be regarded as an evaluation agency of an analyst and a rating company, but it is generally put as an evaluator. An evaluator has the function which receives the cost from a player and makes known characteristic function value in designated coalition to all players. I'll think a little more deeply about an evaluator of composite web service. It is possible to regard as number of accesses of service and number of accesses how much service has value, by the advertisement rate for which we depend on click-though count. But access of service reflects the marketing element which is not the kind. In other words, it influenced by interface whether you put the emphasis and on sale. But, it is not equal to the quality of the service, so the value of the service judged from the user may be the one which is an inevitable problem. This method has a possibility that the system of the back end that the value was difficult to measure can also estimate. For example the user can't use it because the service can't be used when the back end to which how long is catchy service important even if they gather. And therefore evaluation becomes equal to approximately 0. In other words, it is able to become possible by this method in the past to calculate the value of the system of the back end decided by dealings of 1 : 1 and experience. I am thinking assortment of service profit sharing in the service you did some degree fixing of mainly by this research. But the service with which the user can combine service freely is also considered. By what kind of means in cooperative game coalition is made in this case, argument also becomes possible.

3.3 Complements of the Method

In composite web service, players need to calculate imputation, for profit sharing after the whole profit was produced. And we can think the case of profit sharing before players produce the profit, how ratio they will divide the profit. Now because of easiness to deal with, I mainly focus on the case after profit produced. In the problem before profit produced, if we can guess expected profit, we can treat the problem almost same way. But, below discussion I focus on the case after profit produced.

Chapter 4 Proposal of Solution Method

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4.1 Protocol

In the problem setting I explained, I think what procedure is the best to determine imputation. The key is who will evaluate the characteristic function value. Naturally thinking, unknown characteristic function values should be evaluated one by one. It is because to think problem simple. This is, one of the player choose one characteristic function values and pay the cost in one phase. The way to choose what characteristic function values to evaluate is no object now. In this problem setting, players can act by players' knowledge. The knowledge is only the information of characteristic function values which coalition he join. From this knowledge, the players know only other players characteristic function value's upper bound. But player can know his or her imputation's increase when some characteristic function values added to shared knowledge. That is, player can predict how much his or her imputation will increase when a characteristic function value he knows to be evaluated. Then, one of the player's strategies is to calculate increase of imputation for all characteristic function values he or she knows to be added shared knowledge. Think who will get a right to evaluate the particular characteristic function value. For example, when auction style is selected we can choose the player who is the most aggressive player to evaluate. That is, in bidding, we can choose whose increase of imputation is the biggest. Other method, player makes a declaration increase of his or her imputation and maximum player get the right to choose what characteristic function value to evaluate. This has some merit compared to auction. It is simple because of there is no procedure about bidding.

Player makes declaration increase of his or her imputation, player can include his or her thought. For example, if player makes declaration bigger than his or her real increase, his or her probability to get the right will increase than honest. If player makes declaration smaller than his or her real increase, his or her probability to get the right and pay the cost will decrease than honest. But if all players do not join the declaration to avoid paying cost, we want to avoid this situation. If the protocol will stop when no one join some player will make a loss. So, the players whose increase is big, will goes on join the game. I will explain the strategy correspond to this protocol.

Algorithm 1 protocol for imputation

Set each player's knowledge
loop
  Set the characteristic function values which is not include shared knowledge as which satisfies *super-additive*
  Calculate temporal *Shapley* value
  Each player make declaration of his or her value at same time
  if All players make declaration of 0 **then**
    return Temporal *Shapley* value
  end if
  The player made declaration bigger than any other players become pivot player
  if There are several players whose declaration is maximum **then**
    pivot player is chosen randomly from players whose declaration maximum
  end if
  pivot player pay the fixed cost.
  pivot player point the characteristic function value which to be evaluated
  Add the function value to shared knowledge
end loop

I will explain the protocol in this sentence. In initial states, players know only the characteristic function values of the coalition the player join. From this states, protocol starts and reiterate the phase who will pay the cost for making know characteristic function value. In this phase, all player acts according to his or her strategy. And players decide how much he or she will bid to determine who will the pivot player. Pivot player is the highest bidder and pivot player can choose which characteristic function value to be evaluated. At this stage pivot player indicate what characteristic function value to be evaluated, and pay the cost. And the pivot player adds the characteristic function values to all players ' knowledge. If all players bid zero, then the protocol ends. That is, no player joins the game positively. I hope this protocol makes fair imputation by paying cost by player who will increase his or her imputation and update of characteristic function values will close to fair imputation. In next chapter, I will consider the strategy of players on this protocol.

4.2 Strategy

In this section I will consider the strategy and propose the strategy. This is simple but I think the player can reach good imputation compared to all characteristic function values are evaluated.

The action players can select is only declaration and indication of characteristic function value. That is how much player make declaration. I will show player's strategy.

In the protocol I proposed, the only action players can take is making declaration in each phase and, if player became pivot player, choose the characteristic function value to be evaluated. Now, I will organize the information which players have. First, the characteristic function values of the coalition the player join and shared knowledge, the number of players and the value of the cost. As implicitly information, player can know other players upper bound of characteristic function values from *super-additive* and player's knowledge. But player cannot estimate the distribution of characteristic function values and it is difficult to use the information. Other strategy, all time do not join the game to avoid paying cost or joining all the game and reach ideal *Shapley* value instead

Algorithm 2 Player's strategy

s: coalition

$v(s)$: characteristic function value of coalition s

ϕ_i: *Shapley* value of player i

ϕ_i': updated *Shapley* value of player i

n: the number of players

c: cost

K_p: Knowledge of player i, coalition

K_{shared}: Shared knowledge, coalition

L: List which store coalition

L_k: kth coalition which stored in list L

$listL \Leftarrow K_p \notin K_{shared}$

$k \Leftarrow sizeof \quad L$

$max_i \Leftarrow 0$

for $i = 0 \quad to \quad k$ **do**

  $v(s) \Leftarrow 0 : s \notin K_{shared} \cap s \neq s_k$

  calculate ϕ_i'

  $diff \Leftarrow \phi' - \phi_i$

  if $diff > max$ **then**

    $max \Leftarrow diff$

    $s_max \Leftarrow s_k$

  end if

end for

if $max - c < 0$ **then**

  declare 0

end if

if $max - c > 0$ **then**

  return max

end if

if $The player became pivot player$ **then**

  return s_k

end if

of a lot of cost.

Figure 1 shows example procedure, five player and player E's single characteristic function value is biased large. A vertical axis indicates imputation of each player, and horizontal axis indicates phase of protocol. In initial states showed left periphery, we can observe all players' imputation is same, divided equally. Now, the cost equals zero and in right periphery all players reach their *Shapley* value. Viewing this graph, we can know all players ' imputation vary rapidly at first and converge gradually. Next chapter I evaluate the protocol statistically with the computer experiment. Figure 2 shows example procedure, eight players. Each characteristic function values are set as random 0-10 the coalition is single coalition. And other coalition's characteristic function values are set as *super-additive* , the biggest combination of characteristic function values of the coalition's subset coalition, plus 0-10 random. Now set the cost for evaluate as 1.0 and show the example. We can observe in sixteenth phase, the protocol stops. That is, there are 256 characteristic function values, it is we can calculate imputation from 6.25 % of characteristic function values. The right periphery's short line shows ideal *Shapley* value, we can observe it reached almost near value.

Figure 3 shows example procedure, five player and player E's single characteristic function value is biased large. In this time, I plotted evaluation function together. Evaluation function's axis is right. The evaluation function is the sum of squares the difference between output imputation and ideal *Shapley* value. I can observe, instead of its wave, it converges to zero.

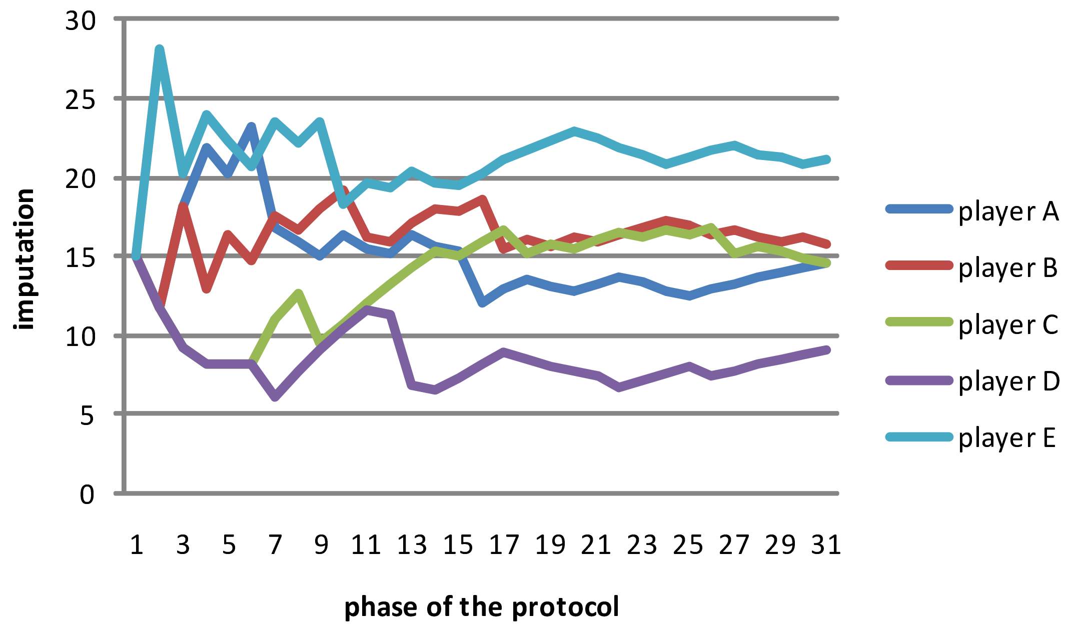

Figure 1: Example of procedure the protocol goes and imputation shift (cost=0)

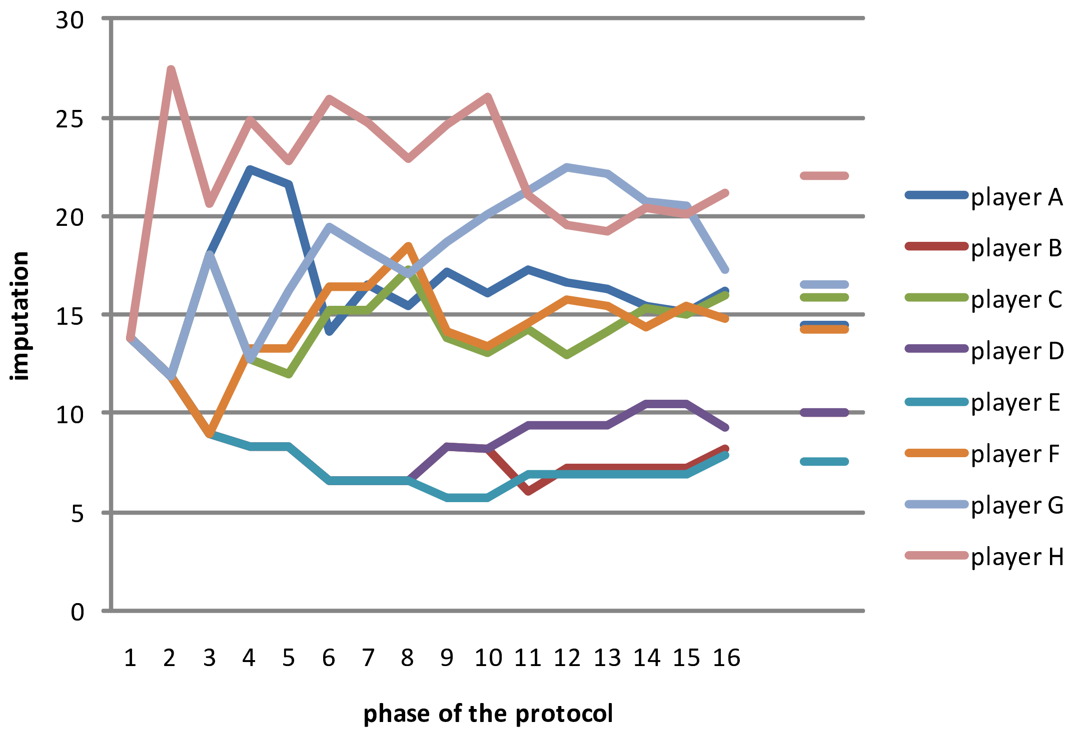

Figure 2: Example of procedure the protocol goes and imputation shift (cost=1)

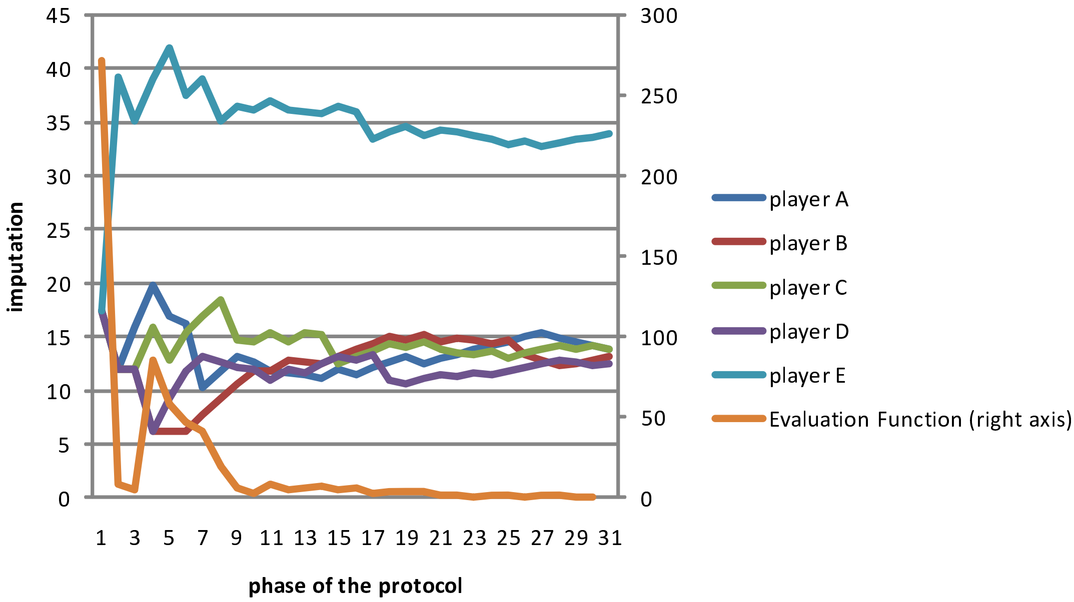

Figure 3: Example of procedure the protocol goes and imputation shift and evaluation function (cost=0)

4.3 Example of sequence

This I will explain example of strategy and protocol goes on.

Table 4: Example: Characteristic function values

coalition	characteristic function value
{A}	8
{B}	5
{C}	3
{A,B}	15
{B,C}	11
{C,A}	14
{A,B,C}	20

Think when three players produced some profit and share the profit with the members. I will take a example problem which coalition and characteristic function values are set like table 4. Now, problem is to divide the grand coalition's profit. As I have discussed in problem setting, this characteristic function values are not necessarily the case that it corresponds to real profit, but it could be expected profit or evaluation of the coalition. For example, if grand coalition produced 30,000 yen, it is not necessarily that 30,000 is characteristic function value. This is only one of the candidates and for availability. To think simply, evaluation by evaluator like user as characteristic function value of the coalition. Then we calculate *Shapley* value and share the profit with the ratio of *Shapley* value. Of course, the characteristic function value of the coalition may vary, and there is not necessarily save the ratio. Then, characteristic function values set as table 4, players knowledge become as table 5 in this problem settings.

Table 5: Example: Knowledge of the players

player A	player B	player C
$v(\{A\}) = 8$	$v(\{B\}) = 5$	$v(\{C\}) = 3$
$v(\{A,B\}) = 15$	$v(\{A,B\}) = 15$	$v(\{A,C\}) = 14$
$v(\{A,C\}) = 14$	$v(\{B,C\}) = 11$	$v(\{B,C\}) = 11$
$v(\{A,B,C\}) = 20$	$v(\{A,B,C\}) = 20$	$v(\{A,B,C\}) = 20$

This means the player knows characteristic function values which coalition he or she joins. To treat in cooperative game theory, I set each players know their characteristic function values strictly, in real it may be players know the characteristic function values vaguely. Then players will act according to this knowledge. And the problem showed on table 4, I assume *super-additive* . This is that the characteristic function value is larger than any subset coalition's characteristic function values. From this assumption player can calculate upper bound of other players' characteristic function values. But I do not think this calculation method because its complexity and we cannot calculate *Shapley* value from the information of ranges of characteristic function values.
In this example the cost for evaluation is zero.

Problem is set as above, let's see how protocol works.

First, list up shared knowledge, which is characteristic function values all player know.

Table 6: Example: Shared knowledge, characteristic function values and its coalition

coalition	characteristic function value
{A,B,C}	20

As table 6, in initial states, the characteristic function values which belongs to shared knowledge is only grand coalition.

Let's calculate *Shapley* value from this shared Knowledge. Now, we assume characteristic function values which do not belongs to shared Knowledge as zero. In first phase, we regard all characteristic function values without grand coalition's as zero.

Table 7: Example: First *Shapley* value, imputation

Player	*Shapley* value
A	6.67
B	6.67
C	6.67

In first phase the imputation becomes as table 7, that is divided equally. Then, this imputation is presented to all players and player will be forced to act. That is how much he or she makes declaration and what characteristic function value to choose.

Now, player acts with his or her strategy. Let's see how player A acts.

If player A make $v(\{A\}) = 8$ evaluated, he or she calculate temporary *Shapley* value with shared knowledge showed table 6 plus $v(\{A\}) = 8$. Then imputation become as table 8.

Table 8: Example: Imputation when A evaluated $v(\{A\}) = 8$

player	*Shapley* value
A	12
B	4
C	4

This mean player A's profit increase 20/3 to 12. As this, I calculate what characteristic function values player A evaluate from his or her knowledge and how much his or her profit will increase as table 9.

Table 9: Example: First calcuration of imputation of player

characteristic function value to evaluate	*Shapley* value
$v(\{A\})$	12
$v(\{A,B\})$	55/6
$v(\{A,C\})$	54/6

From these characteristic function values, the characteristic function value which bring the largest increase for player A is $v(\{A\})$. And then, the increase of player A's profit is $12 - 20/3$. I call the increase as player A's maximum expected increase of profit. This player A's maximum expected increase of profit is larger than fixed cost 1.0, player A goers on the game. If this player A's maximum expected increase of profit is smaller than fixed cost, player A will make a loss when he or she pay the cost for evaluation. So, player A will not go on the game.

Each player make declaration of each maximum expected increase of profit, and maximum player get a right to evaluate the characteristic function value he or she choose.

Calculation of other players' maximum expected increase of profit become as table 10.

Table 10: Example : maximum expected increase of profit of each players

player	maximum expected increase of profit	coalition
A	$12 - 20/3$	$v(\{A\})$
B	$10 - 20/3$	$v(\{B\})$
C	$9 - 20/3$	$v(\{A,C\})$

From this, the player whose maximum expected increase of profit is larger than any other players is A. Then player A pay the fixed cost 1.0 and make evaluated the characteristic function value $v(\{A\})$ and add this to shared knowledge.

Then, shared knowledge is updated as table 11.

Table 11: Example : Shared knowledge, characteristic function values and its coalition

coalition	characteristic function value
{A}	8
{A,B,C}	20

In this states, each players profit is calculated as table 12.

Table 12: Example: First *Shapley* value, imputation

player	*Shapley* value
A	12
B	4
C	4

We iterate these procedures till all players' maximum expected increase of profit become smaller than the fixed cost.

Chapter 5 Evaluation

In this strategy, I do not consider players' mutual strategic situation a little. If a player evaluate the characteristic function values that also bring increase of the player, he or she will avoid to make evaluate the characteristic function value with the cost. This indicates if cost could be varied, then strategic situation will come in.

5.1 Intuitive Consideration

In this section I will explain the merit and properties of this protocol and strategy.

The players' action on this protocol is only to make declaration on each phase. Then, I will think how much the player should choose.

1. When player make declaration bigger than his expected profit increase Now, the player declared bigger is tend to be selected, and then his profit will increase in the immediate term. On the other hand, other players will act after the player act, the player may pay unnecessary cost. It may be the player's loss.
2. When player make declaration smaller than his expected profit increase In this case, the cost player pays will decrease. If only the player pay small, the player will get a bigger profit because the player do not pay the cost appropriate. But, if all players act this strategy, extreme case, all players quit game, and protocol will stop. Then the player who had contributed a lot and expected to get a lot, may have dissatisfaction or excess.

If all players not make declaration and wait other players pay the cost, then the protocol stops. Then the player who had contributed a lot and expected to get a lot, may have dissatisfaction. So, the player who had contributed a lot and expected to get a lot have to go on to join the game. That is, this protocol has a structure which the player who had contributed a lot and expected to get a lot, cover the cost.

This is why there are players whose appropriate profit, the *Shapley* value is smaller than the profit divided equally in initial state. And the player who had contributed a lot and expected to get a lot pays the cost for their appropriate imputations.

This protocol is natural and simple on the problem setting. This protocol has property that stop without fail, but the expected properties I explained section 2.4 are not necessarily guaranteed. This is because, the attribute of problem setting that there is indeterminacy brought by random and asymmetrical information. To confirm this protocol's usefulness, I carried out some experiments in next chapter.

5.2 Computer Experiment

I have done computational experiment to evaluate the protocol and strategy I proposed. I made combinations of players and characteristic function values and operate the protocol. And I observed the shift of imputation.

5.2.1 Experiment Settings

I consider the situation which six players and sixty-four coalition. A single coalition's characteristic function value is determined 0 to 10 randomly. The Other coalitions are determined as which satisfies *super-additive* , that is I added 0 to 10 randomly on maximum combination of subset coalition's characteristic function values.

Now I have done two experiments, first, how the variance of the cost influence on evaluation function which represent players' fair imputation. Second, how the bias on the characteristic function values influence on the procedure of the protocol.

The evaluation function is the square sum of difference between output imputation and ideal *Shapley* value.

1. Experiment 1

   I plot evaluation function on the graph, with varied cost 0 to 1.5 by 0.5 step in same distribution of the characteristic function values.

2. Experiment 2

   I plot evaluation function with varied distribution of characteristic function values when cost is fixed. The distribution of characteristic function values are no bias, weak bias and strong bias. The weak bias means one player's single coalition's characteristic function value is twice bigger than others, and strong is three times multiplied.

In each experiment I have experimented a thousand, and use average data.

5.2.2 Hypothesis

1. Experiment 1

   If cost is bigger, then protocol will stop soon, but the evaluation function is not improved as a case the cost is smaller. I predict if cost increase, then average number of phase till stop will increase exponentially.

2. Experiment 2

If there is no bias in characteristic function values, the initial evaluation function is rather good than there is some bias. Because in the situation when there is no bias. Dividing equally is good imputation. That is if there is some bias, evaluation function will be improved compared to not. But an average number of phase till stop will grow.

5.2.3 Results

1. Experiment 1

   We plot experiments with four types of cost when there is no bias in characteristic function values on figure 4. The figure 5 shows the relation of evaluation function when there is weak characteristic function value. The vanishing point means the protocol stops. When the cost = 0, the protocol do not end till all sixty-four characteristic function values are evaluated.

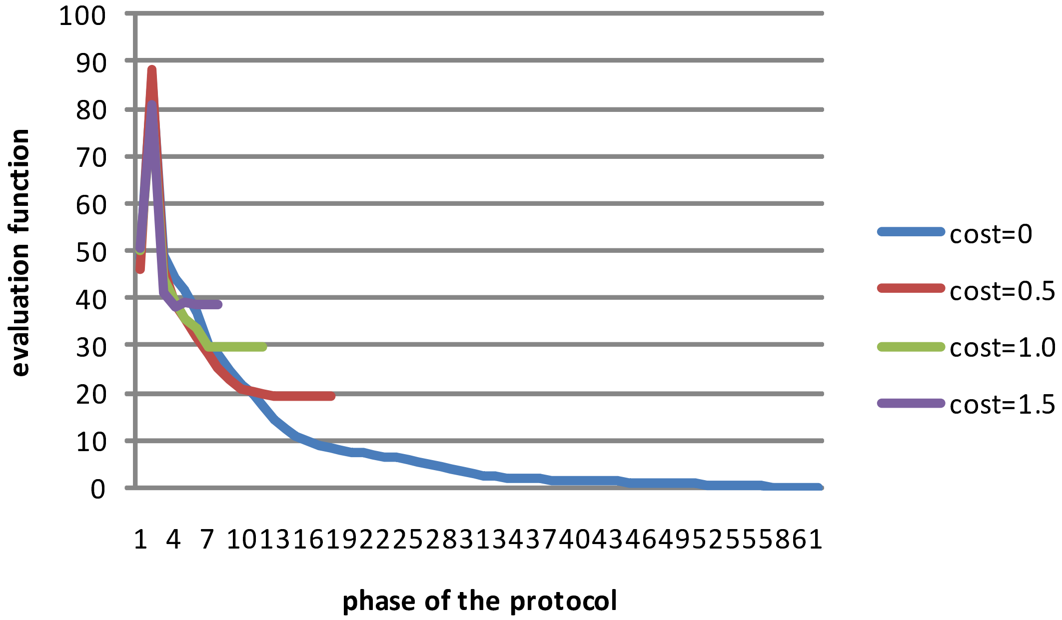

Figure 4: Relation in cost and evaluation function. when there is no bias

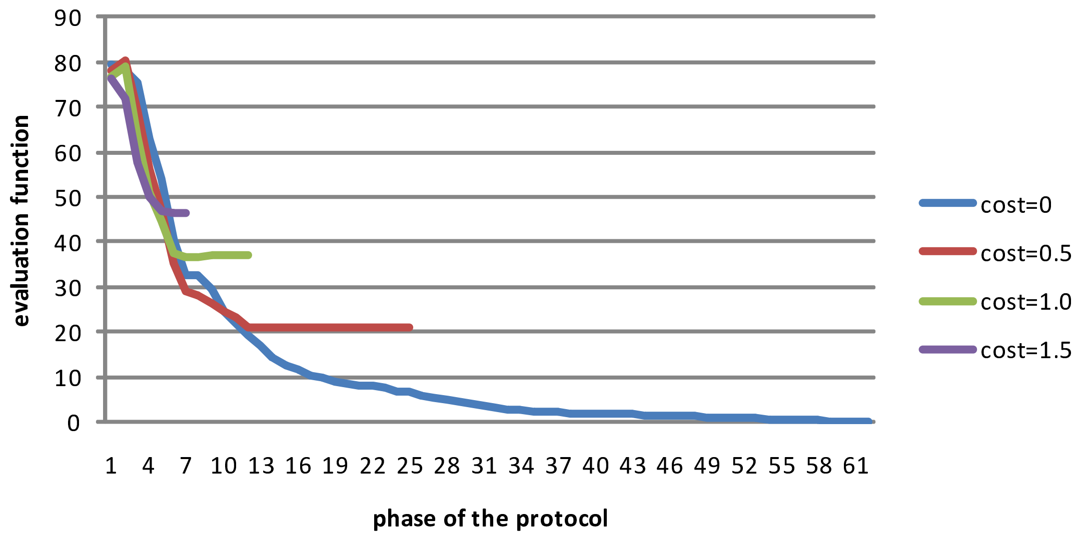

Figure 5: Relation in cost and evaluation function. when there is weak bias

Next we show average number of phase till stop the protocol, table 13 shows when there is no bias in characteristic function values. A table 13 shows when there is weak bias in characteristic function values. A table 13 shows when there is strong bias in characteristic function values.

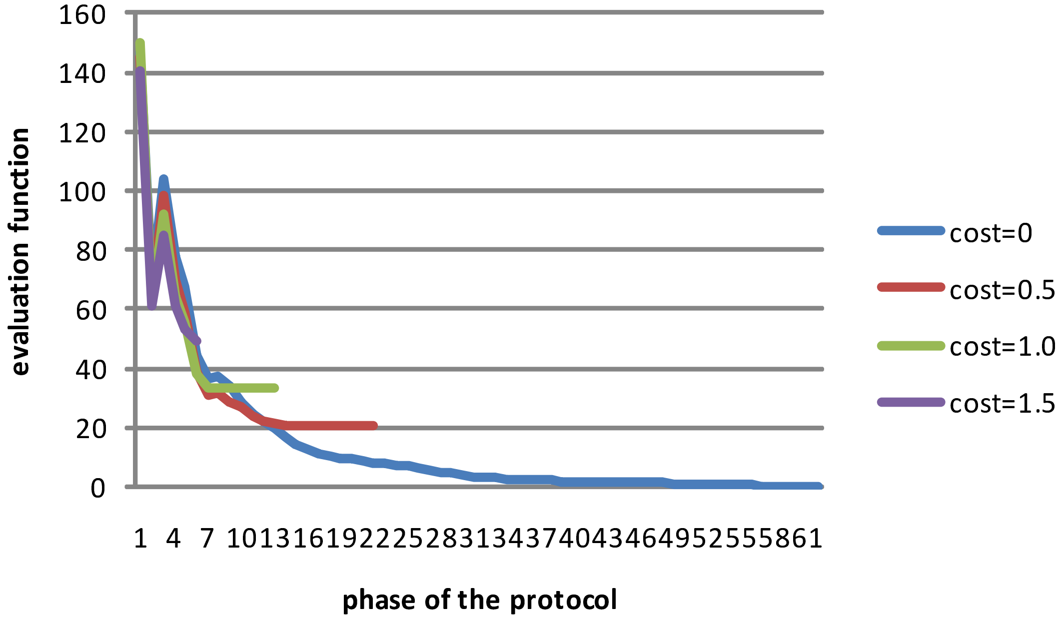

Figure 6: Relation in cost and evaluation function. when there is strong bias

Table 13: Relation between cost and evaluation function when there is no bias.

condition	average number of phase till stop
cost=0	62.00
cost=0.5	18.49
cost=1.0	10.35
cost=1.5	7.15

Table 14: Relation between cost and evaluation function when there is weak bias.

condition	average number of phase till stop
cost=0	62.00
cost=0.5	19.18
cost=1.0	10.91
cost=1.5	7.46

Table 15: Relation between cost and evaluation function when there is strong bias.

condition	average number of phase till stop
cost=0	62.00
cost=0.5	19.51
cost=1.0	11.08
cost=1.5	7.64

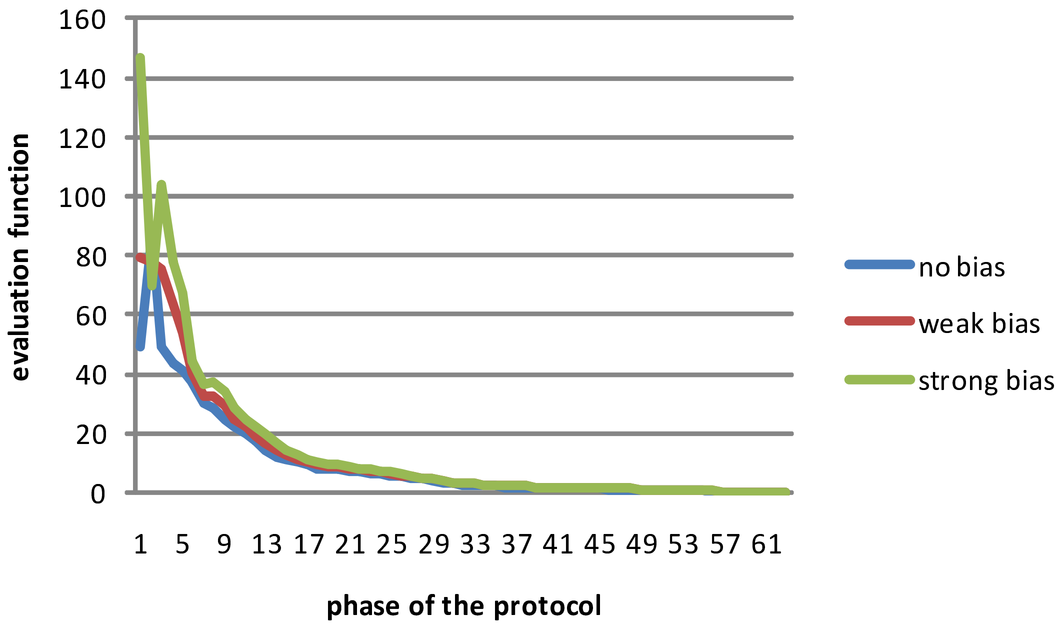

Figure 7: Relation of average number of phase till stop and evaluation function : cost=0

2. Experiment 2

I experimented procedure of protocol and evaluation function when there are bias in characteristic function values. There are four version as cost 0, 0.5, 1.0, 1.5. A figure 7 shows when cost = 0. A figure 8 shows when cost = 0.5. A figure 9 shows when cost = 1.0. A figure 10 shows when cost = 1.5.

Next, we show the average number of phase till stop when figure 9 and

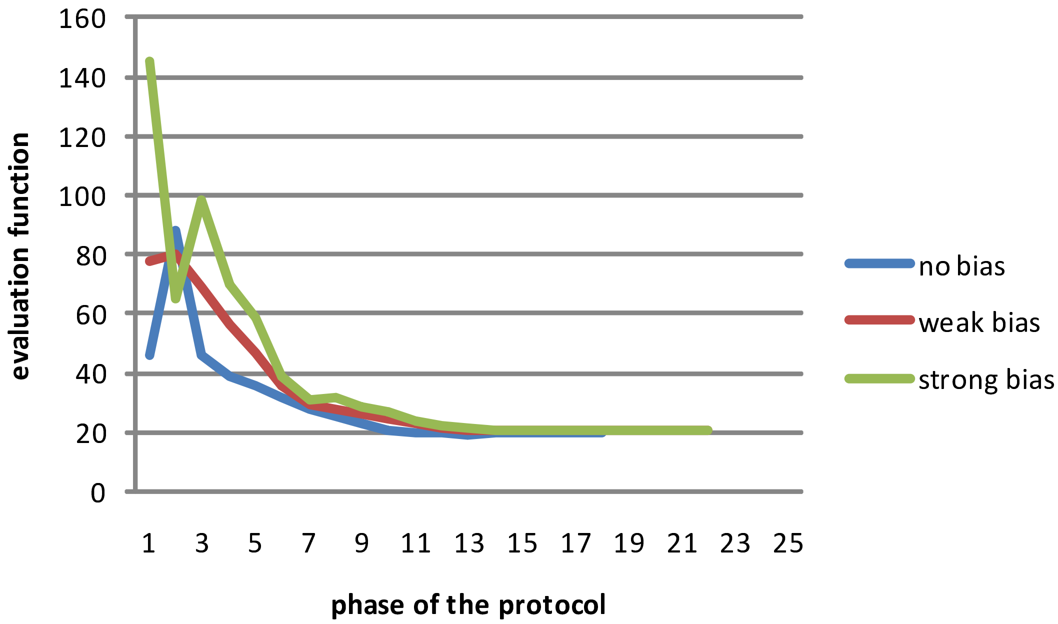

Figure 8: Relation of average number of phase till stop and evaluation function : cost=0.5

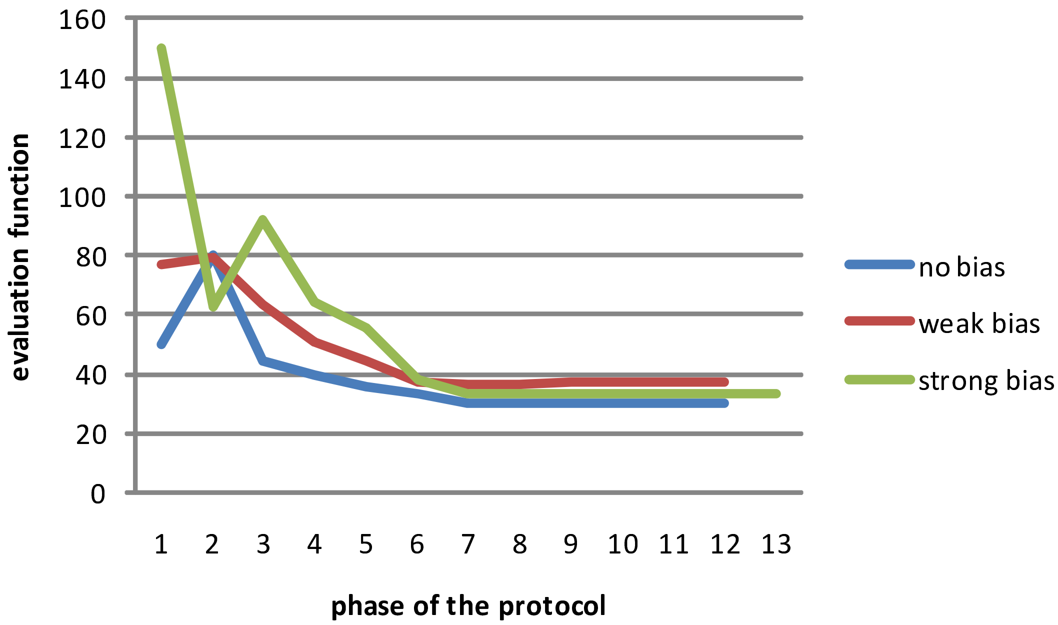

Figure 9: Relation of average number of phase till stop and evaluation function : cost=1.0

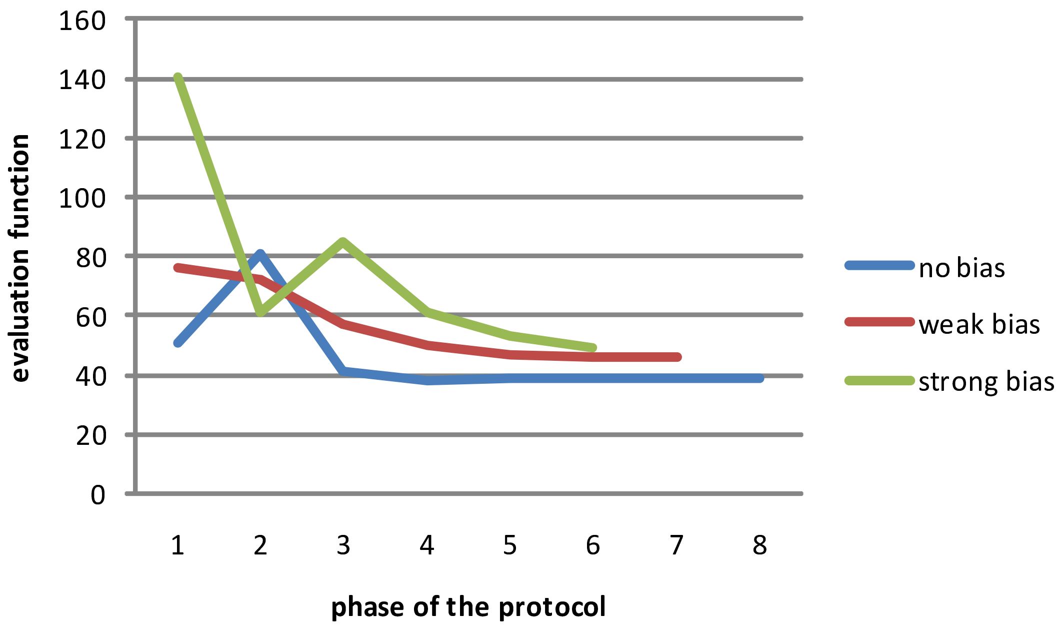

Figure 10: Relation of average number of phase till stop and evaluation function : cost=1.5

Table 16: Relation of average number of phase till stop and evaluation function when the distribution of characteristic function values are biased, cost = 0.5

condition	average number of phase till stop
no bias	18.49
weak bias	19.18
strong bias	19.51

figure 10. I do not show the graph when cost = 0, because the protocol goes on till all characteristic function values are evaluated.

And table 19, table 20, table 21 shows conclusive evaluation function, if the protocol do not stop when we varied cost 0.5, 1.0, 1.5.

5.2.4 Consideration

1. In experiment 1, we can see evaluation function become worse rapidly at beginning of the protocol when there is no bias in characteristic function values from figure 4. It is conceivable that, when there is no bias in characteristic function values, average players' ideal profit is close to divided

Table 17: Relation of average number of phase till stop and evaluation function when the distribution of characteristic function values are biased, cost = 1.0

condition	average number of phase till stop
no bias	10.35
weak bias	10.91
strong bias	11.08

Table 18: Relation of average number of phase till stop and evaluation function when the distribution of characteristic function values are biased, cost = 1.5

condition	average number of phase till stop
no bias	7.152
weak bias	7.476
strong bias	7.644

Table 19: conclusive evaluation function if all characteristic function values are evaluated

condition	conclusive evaluation function
no bias	175.1
weak bias	174.8
strong bias	174.9

Table 20: conclusive evaluation function if all characteristic function values are evaluated

condition	conclusive evaluation function
no bias	700.1
weak bias	699.3
strong bias	700.5

Table 21: Conclusive evaluation function if all characteristic function values are evaluated

condition	conclusive evaluation function
no bias	1570.9
weak bias	1571.7
strong bias	1577.8

equally, and when specific characteristic function value was evaluated it will spread from appropriate imputation. But, once the evaluation function take worse, then we can see it improved smoothly and stop with good value compared to initial state. Because of buildup of cost players paid conclusive imputation, conclusive imputation is not equals ideal *Shapley* value and evaluation does not equals to zero. It is why the protocol stops soon when the fixed cost is big, and evaluation function is remote from zero as we can see on table 13. We can see evaluation function is improved rapidly when there is a bias in characteristic function values compared to not, from figure 5 and figure 6. But, if there is strong bias in characteristic function values, the evaluation function is improved after once the function become worse. It is conceivable that a player whose characteristic function value is biased large get good imputation at a breath and swung over.

2. Seeing result of experiment 2, we can know evaluation function's shift is very similar whichever the cost, at beginning. That is, when there is no bias, worse at beginning and improved soon, when there is weak bias, not vary at beginning and improved rapidly, when there is strong bias, initial evaluation function is worse but improved rapidly and once become worse and improved again.

   Seeing table 19, table 20, table 21, we can know when the cost is bigger, the conclusive imputation if protocol goes on last is worse. Clearly this is worse than initial state or processed imputation, we can know the protocol has improved the imputation.

   And it is interesting than in each case protocol stops at almost same time, instead of its bias.

And table 13, table 14, table 15 shows protocol stops regardless of its characteristic function values bias. This indicates we can predict a number of stops according to the cost. We have to check this with additional experiment.

I will marshal the things I can confirm in these experiments.

1. In the range of cost I experimented, we can reach good imputation regardless of the bias of characteristic function values or cost compared to initial situation divided equally or ideal Shapley value evaluated all characteristic

function values.

2. In the range of cost I experimented, average number of phase till stop is almost same regardless of the bias.

I make a supplementary statement about first point. When the cost is zero, conclusive imputation reach *Shapley* value, and with the set of cost, the conclusive imputation may become good than initial, divided equally imputation. But in such case the imputation by the protocol will be good than the case, and I will not consider any longer.
Second, what an average number of phases till stop will be have to be researched more. And I will run additional experiment how an average number of phase till stop become regarding to characteristic function values' bias.

5.2.5 Third Experiment

From above discussion, the hypothesis the number of phases protocol stops is depends on the cost was indicated. So, in this section, I will confirm the hypothesis.

When there are five players and thirty two characteristic function values, I varied the cost 0 to 1.75 by 0.25 steps comparing four type of biased characteristic function values. I have calculated 1000 and save the frequency of a number of phases the protocol stops.

Figure 11 shows when cost equals 0.25, figure 12 is cost 0.5, figure 13 is cost 0.75, figure 14 is cost 1.0. Figure 15 shows relation of average took phases till protocol stops and the cost when there is no bias.

Seeing figure 11, it seems when the cost is small there is erratic pattern. It is because, when the cost is small it takes a long time till stop the protocol. In each figure, we can see when the characteristic function values are biased, it takes a bid longer than not. This is natural. And when cost is over 0.5, we can observe frequency of the phase the protocol stops shows normal distribution. This is good property because of it shows protocol's stability.

And from the figure 15 we can observe the average taken phases till protocol stops logarithmically decrease. This is not the property I have wanted to show at first, but this is good property which we can predict needs of the phases till stop the protocol.

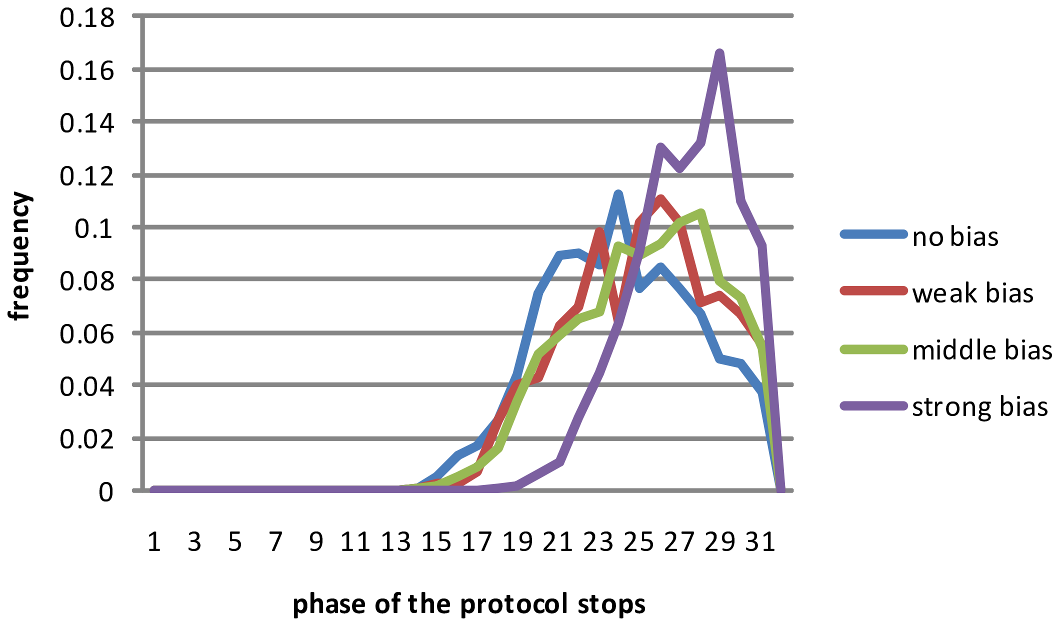

Figure 11: Frequency of the phase the protocol stops : cost=0.25

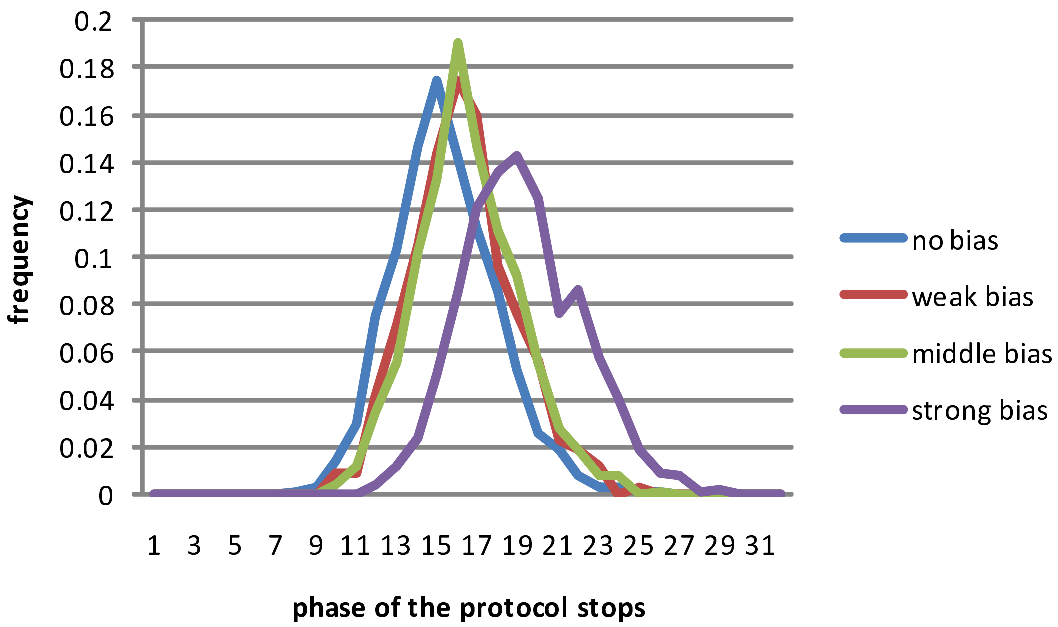

Figure 12: Frequency of the phase the protocol stops : cost=0.5

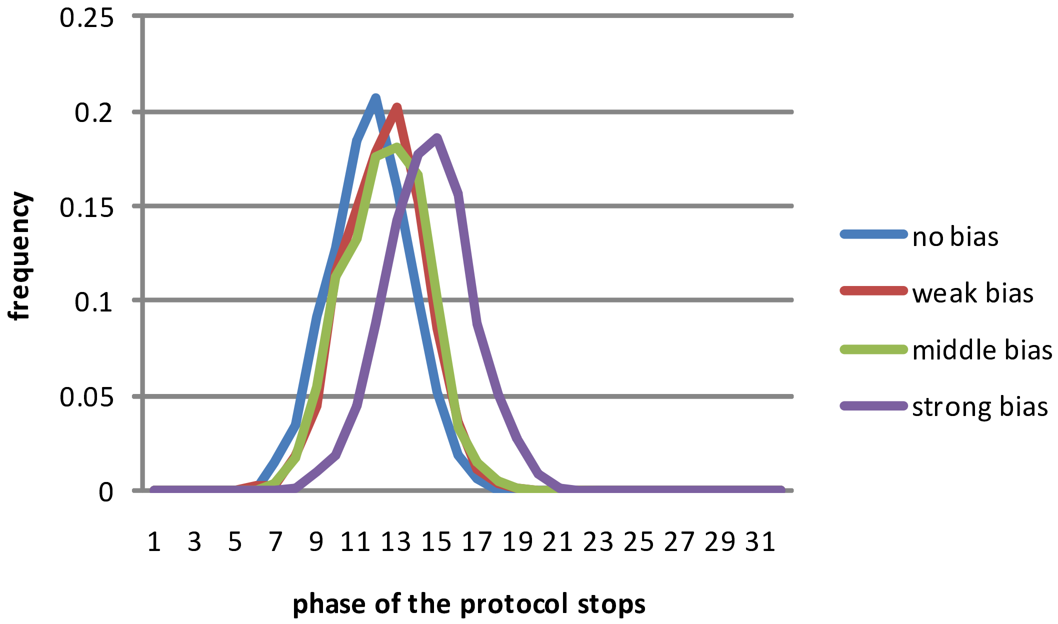

Figure 13: Frequency of the phase the protocol stops : cost=0.75

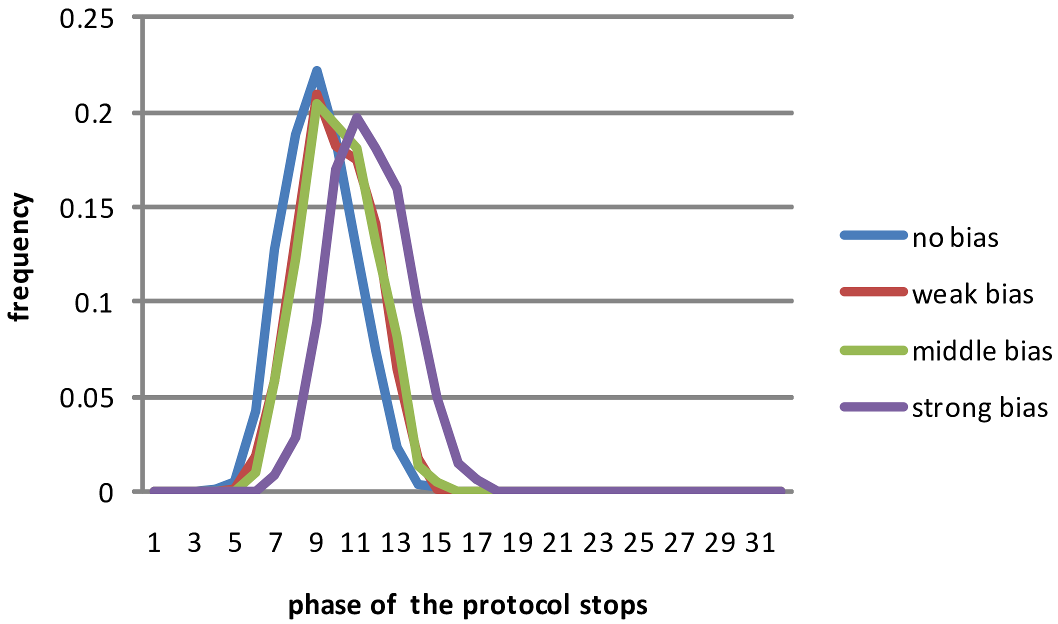

Figure 14: Frequency of the phase the protocol stops : cost=1.0

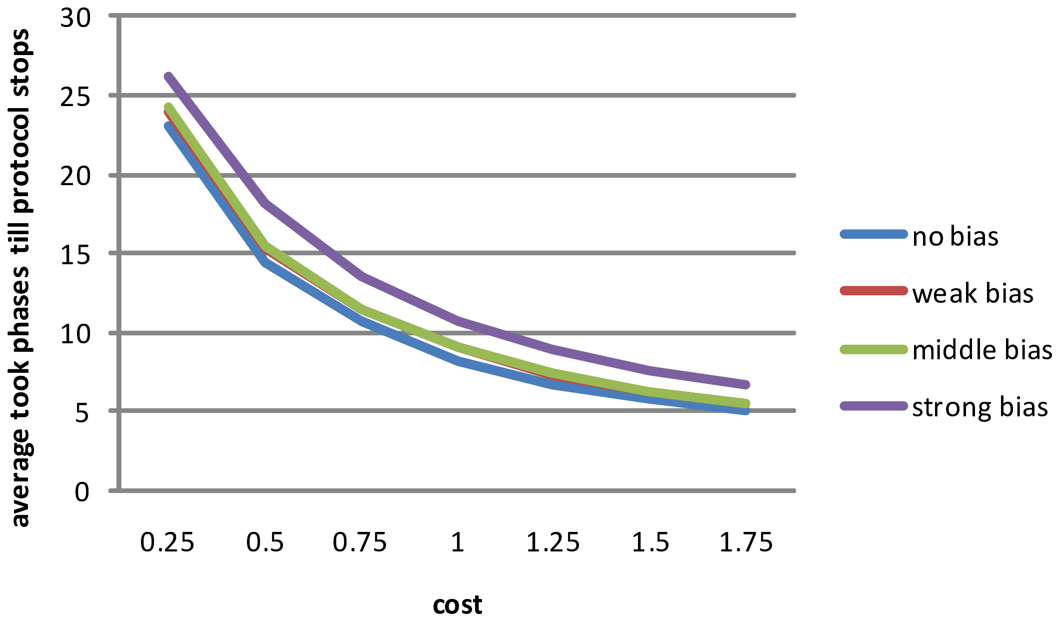

Figure 15: Relation between average taken phases till protocol stops and cost

Chapter 6 Conclusion

In this research, I have considered solution for profit sharing problem. I focused on the problem with incomplete information about characteristic function values, but we can know the value with the cost to evaluate the value. There is no sufficient discussion to treat the problem. In such situation I proposed problem settings, players' knowledge and way to evaluate unknown characteristic function values. Then, I indicated the strategy of the players to reach fair imputation on the protocol. I suggested it is one of the best strategies for player to act to increase his imputation in the immediate term. Because of its indeterminacy, it is difficult to discuss the properties of the protocol in a precise sense. But I checked the property by several experiments, and the protocol can improve the imputation. However it is not guarantee of global optimum. In this research, I considered the problem which we cannot treat with traditional theory of cooperative game. In this problem setting, we can treat the problem with unknown characteristic function values. This will contribute to spread field of theory of cooperative game. And this will enable to apply real problem which we cannot treat before.

In this research, I solved the problems as below.

1. I formalized the problem settings when there are unknown characteristic function values but we can know characteristic function values with the cost.
2. I proposed a protocol to reach fair imputation in the situation.
3. I proposed a method to calculate the *Shapley* value when there are unknown characteristic function values.

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