Master Thesis

Analyses of an Internet Auction Market Focusing on the Fixed-Price Selling at a Buyout Price

Supervisor Associate Professor Shigeo Matsubara

Department of Social Informatics
Graduate School of Informatics
Kyoto University

Hiromichi ARAKI

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Hiromichi ARAKI

Abstract

Internet Auction is a one of the most successful e-Commerce markets. Recently, it has been reported that the trades having buyout options are increasing. A "buyout option" is available in many Internet auction sites. When a seller uses a buyout option, a "buyout price (buy price)" of the good is set by the seller. If a buyer submits a bid equal to the buyout price, the auction immediately ends and the buyer can obtain the good by paying the buyout price. If the seller sets the start price to the price equal to the buyout price, it can be viewed as fixed-price selling.

In the recent auction sites, identical goods are sold in an auction with a buyout price and in an auction without a buyout price simultaneously. Considering such a situation, understanding how a buyout option affects the market is significant to design the future auction markets. However, there are following two problems.

Understanding sellers' behaviors in an Internet auction market As a first step to understand the effect of a buyout option, we must know the real situation in an Internet auction market. In particular, understanding sellers' behaviors in auctions with a buyout option is required. Therefore, we need to understand seller's behaviors in the actual market.

Building a model based on the situation in actual markets Previous studies have mainly focused on clarifying the conditions which selling format outperforms. In the actual Internet auction market, the both types of sellers using the buyout option and not using the buyout option simultaneously exist. However, researchers have paid little attention to the interaction between the two selling formats. Therefore, building a model to explain the situation in the actual market is required.

In this research, in order to solve the above problems, the author has characterized the major seller's behavior by analyzing the actual auction data and proposed a model including two sellers and three buyers.

For the first problem, the author analyzes the actual data in an Internet auction market. In particular, the seller's behavior is analyzed by focusing the start prices and buyout prices which sellers set.

For the second problem, the author proposes a model of an auction market with a buyout option where two sellers exist considering the result of data analysis. First, the case where sellers' strategies are limited to the major strategies obtained from the actual data is discussed. Secondly, the strategies in the perfect Bayesian Nash equilibrium are compared to the strategies observed in the actual data.

The contributions of this research are summarized as follows.

Presenting major strategies of sellers in an Internet auction market 11,921 auction data obtained from an actual Internet auction site were examined by focusing on the setting of start price and buyout price. The results of data analysis show the two major strategies of the sellers in the market as follows: (1) many of sellers who set buyout prices sell by fixed-price selling at a buyout price, (2) many of sellers who do not set buyout prices set start prices at quite low price.

Proposing the model to explain the coexistence of two type sellers The author can successfully provide a model able to explain the situation where the both types of sellers using the buyout option and not using the buyout option simultaneously exist. The model supposes a two-stage game where two sellers arrive sequentially. First, the case where the seller's strategy is restricted to the two strategies obtained from the actual data was discussed. In this case, if the probability that a buyer is risk-averse is quite high, both two sellers can benefit by selling a good by using a buyout option and selling another good by using an ascending auction. Secondly, the strategies in the perfect Bayesian Nash equilibrium are showed. If the first seller has a large valuation to his good, the combination of the strategies in the equilibrium corresponds to the two major strategies in the actual data. When sellers select strategies satisfying the perfect Bayesian Nash equilibrium, the total revenue of the sellers is higher than the case no seller sets a buyout price.

即決価格による固定価格販売に着目したインターネットオークション市場の解析

荒木 博道

内容梗概

インターネットオークションは最も成功した電子商取引の一つであるが，最近になって，オークションで手間をかけずに，素早く財を購入できる固定価格販売による取引が増加する傾向にあると報告されている．オークションにおいて，固定価格販売を実現するオプションとして “即決価格オプション (buyout option)” がある．“即決価格 (buyout price, buy price)” は売り手によって設定され，買い手が即決価格での入札を行えば，オークションを即終了させ，その価格で財を購入することができる．売り手が開始価格と即決価格を同額に設定することで，実質的な固定価格販売となる．

現在のインターネットオークション市場では，即決価格を用いた販売と即決価格を用いない競り上げオークションによる販売の 2 種類が混在する．このような状況において，現実の市場に対する即決価格の影響を理解することは，今後のオークション市場の設計を考える上で極めて重要である．しかし，そのためには，次の 2 つの課題が存在する．

現実のインターネットオークション市場における売り手の行動の理解　市場に対する即決価格の影響を考えるためには，実際のインターネットオークション市場において即決価格の利用の現状を知ることが不可欠である．特に，即決価格オプション使用や即決価格の設定が，実際にどのように行われているかを知ることが求められる．そのためには，市場における売り手の行動を理解することが必要となる．

現実の市場の状況を考慮したモデルの構築　過去の即決価格を伴うオークションに関する研究では，1 人の売り手が財を販売する際に，与えられた状況において，即決価格を使用するべきか否か，あるいは即決価格をいくらに設定すべきかが議論されてきた．しかし，現実のオークションには，即決価格を使用して財を販売する売り手が存在する一方で，即決価格を設定せずに競り上げオークションによる販売を行う売り手も存在する．過去の研究では，このような 2 つの販売方法が互いにどのように影響しあうかについて議論されていない．よって，このような現実の市場の状況を考慮したモデルを構築することが必要となる．

本研究では，上記の課題を解決するために，実データの解析から売り手の行動の特徴を示し，その結果を踏まえて 2 人の売り手が存在する状況のモデル化を行い，即決価格による固定価格販売と即決価格を利用しない競り上げオークションによる販売が混在する市場の解析を行う．

まず第 1 の課題に対して，インターネットオークション市場における実際の取引データについて，売り手の開始価格と即決価格の設定に焦点を当て，売り手の行動を解析する．

次に第 2 の課題に対して，即決価格オプションありのオークションについて，2 人の売り手が参加するモデルを提案する．まず，売り手の戦略を実データから得られた主要な戦略に限定した場合を議論する．さらに，売り手の戦略空間を拡張したモデルにおけるベイズ完全均衡戦略を示し，データから得られた主要な戦略とモデルにおける均衡戦略を比較する．

本研究の主な貢献は以下の 2 点である．

インターネットオークション市場における売り手の主要な販売戦略の提示 売り手の行動を理解するため，実際の即決価格オプションを伴うインターネットオークションにおける 11,921 件の取引データについて，売り手の開始価格設定と即決価格設定に着目した解析を行った．データの解析結果から，市場における売り手の主要な戦略が次の 2 つであることが示された．(1) 即決価格を設定する売り手の多くが，実質的な固定価格販売を行う．(2) 即決価格を設定せず競り上げオークションで販売する売り手の多くが，非常に低い開始価格を設定する．

2 種類の売り手が混在する状況を説明するためのモデルの提案 現実の市場において即決価格を使用する売り手と使用しない売り手の 2 種類の売り手が混在する状況を説明するために，2 人の売り手が存在するオークションモデルを提案した．まず，売り手の戦略をデータから得られた主要な 2 つの戦略に限定した状況を考えた．このとき，即決価格を設定しない場合よりも高い収入を 2 人の売り手が得ることができる状況は，買い手がリスク回避型である確率が高い場合に限定されることが示された．次に，売り手の戦略空間を拡張し，ベイズ完全均衡となる戦略を示した．先に販売する売り手の自己の財に対する評価値が高い場合，データで見られた主要な 2 つの戦略を選択することが均衡となる場合が存在した．また各売り手が，ベイズ完全均衡となる価格設定を行った場合，即決価格が設定されない場合よりも，売り手収入の総和は高くなることが示された．

Analyses of an Internet Auction Market
Focusing on the Fixed-Price Selling at a Buyout Price

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Chapter 1 Introduction

Internet Auction is a one of the most successful e-Commerce markets. Recently, it has been reported that the trades having buyout options are increasing [1]. A "buyout option" is available in many Internet auction sites. When a seller uses a buyout option, a "buyout price (buy price)" of the good is set by the seller. If a buyer submits a bid equal to the buyout price, the auction immediately ends and the buyer can obtain the good by paying the buyout price. If the seller sets the start price to the price equal to the buyout price, it can be viewed as fixed-price selling.

In the research field of agents, there are many studies of auctions [2, 3]. In particular, auctions are adopted in resource allocations of agents [4, 5]. Introducing a buyout option has an advantage that buyers do not need to monitor the situation of bidding if they purchase goods at buyout prices, keeping the procedure easy to understand for humans. This advantage is also useful to auctions by software agents.

In the recent auction sites, identical goods are sold in an auction with a buyout price and in an auction without a buyout price simultaneously. Considering such a situation, understanding how a buyout option affects the market is significant to design the future auction markets.

However, there are following two problems.

Understanding sellers' behaviors in an Internet auction market As a first step to understand the effect of a buyout option, we must know the real situation in an Internet auction market. In particular, understanding sellers' behaviors in auctions with a buyout option is required. Therefore, we need to understand seller's behaviors in the actual market.

Building a model based on the situation in actual markets Previous studies [6, 7, 8, 9] have mainly focused on clarifying the conditions which selling format outperforms. In the actual Internet auction market, the both types of sellers using the buyout option and not using the buyout option simultaneously exist. However, researchers have paid little attention to the interaction between the two selling formats. Therefore, building a model to explain the situation in the actual market is required.

In this research, in order to solve the above problems, the author presents the major seller's behavior by analyzing the actual auction data and proposes a model including two sellers and three buyers. The author analyzed an Internet auction market where ascending auction and fixed-price selling simultaneously exist.

For the first problem, the author analyzes the actual data in an Internet auction market. In particular, the seller's behavior is analyzed by focusing the start prices and buyout prices which sellers set.

For the second problem, the author proposes a model of an auction market with a buyout option where two sellers exist considering the result of data analysis. First, the case where sellers' strategies are limited to the major strategies obtained from the actual data is discussed. Secondly, the strategies in the perfect Bayesian Nash equilibrium are compared to the strategies observed in the actual data.

The rest of this paper is organized as follows. Chapter 2 describes the buyout option and the related researches about it. In Chapter 3, the actual auction data is analyzed. Chapter 4 proposes the model considering the major strategies obtained from the actual data. Chapter 5 extends the model and considers the strategies in the perfect Bayesian Nash equilibrium. Chapter 6 discusses the sellers' behaviors by comparing the result of the data analysis to the result of the analysis of the model. Finally Chapter 7 concludes this paper.

Chapter 2 Buyout Price

A buyout option is one of the options used in Internet auction sites. When a seller uses the option, the seller sets a buyout price of his good in addition to a start price. When a buyer bids at the buyout price, the auction quickly ends and the buyer can purchase the good at the buyout price. For example, a buyout option is used in Yahoo! JAPAN auction[1)] , eBay[2)] and many other Internet auction sites.

The exact nature of the buyout option differs across auction sites. Such options can be broadly characterized as either “permanent” or “temporary”. A permanent buyout option is available for the entire duration of the action, whereas a temporary buyout option may cease to be available before the conclusion of the auction [8]. For example, “Buy Price (*Sokketsu Kakaku* in Japanese)” in Yahoo! JAPAN auction corresponds to a permanent buyout option. On the other hand, “Buy It Now” option in eBay corresponds to a temporary buyout price option. This paper considers the auctions with a permanent buyout option.

2.1 Fixed-Price Selling at a Buyout Price

Setting a buyout price equal to the start price corresponds to selling at a fixed-price in auctions. Therefore, in the recent Internet auctions, there are two selling types: (1) auction without a buyout price, (2) auction with a buyout price. Auctions corresponding to (2) can be further divided into the following two types: (2-a) auction with a buyout price higher than the start price and (2-b) auction with a buyout price equal to the start price (fixed-price selling).

It has been reported that the buyout-option trades are increasing [1]. Understanding how a buyout option affects the market is significant to design the future auction markets.

1) http://auctions.yahoo.co.jp/
2) http://www.ebay.com/

2.2 Related Researches

This section describes the related researches about auctions with a buyout option. Buyout options are noted firstly by Lucking-Reiley [10]. He notes that buyout options allow the bidder to buy an early end to the auction by submitting a sufficiently high bid. Budish et al. shows that a seller's revenue is improved by setting buyout price when a risk-averse buyer exists [6]. The research of Hidvegi et al. shows that social utility is improved by setting the appropriate buyout price in English auctions with permanent buyout price [7]. They have analyzed the case where a seller is risk-averse or buyers are risk-averse. On the other hand, the paper of Mathews et al. have discussed "Buy-It-Now" option in eBay [8]. They have analyzed the case where a seller is risk-averse and buyers are risk-neutral. Reynolds et al. have discussed the two major buyout options: "Buy-It-Now" in eBay and "Buy Price" in Yahoo! JAPAN auction [9]. They have analyzed the case where a seller and two risk-averse buyers exist.

Previous studies have mainly focused on clarifying the conditions which selling format outperforms. For example, the model of Hindvegi et al. elucidates the conditions when the seller should use a buyout option and how to calculate the optimal buyout price [7]. However, researchers have paid little attention to the interaction between two selling formats. In auction sites, identical goods are sold in an auction and at a fixed-price simultaneously. The previous studies cannot explain this situation. Therefore, the author develops a model to explain this situation.

Chapter 3 Analysis of the Auction Data

As a first step to understand the effect of a buyout option, we must know the real situation in an Internet auction market. In particular, understanding sellers' behaviors in an actual market with a buyout option is required. This chapter analyzes sellers' behaviors by using the actual auction data in an Internet auction market with a buyout option. The author particularly focuses on the setting of buyout prices.

3.1 Definition of Indexes

In an actual Internet auction market, the final prices of the auctions widely differ from the types of items. Therefore, the author introduces the indexes to treat many data of multiple items.

At first, define μ_{ij} as the average of the final prices in the auctions about item i in term j. The indexes of start price, buyout price and final price are defined as follows:

$$P_{start} = (x - \mu_{ij})/\mu_{ij},$$
$$P_{buyout} = (y - \mu_{ij})/\mu_{ij},$$
$$P_{final} = (z - \mu_{ij})/\mu_{ij}$$

where x is the amount of the start price, y is the amount of the buyout price and z is the amount of the final price in the one of the auctions about the item i in the term j. In this case, the bound

$$-1 < P_{start} \leq P_{final} \leq P_{buyout}$$

is satisfied.

For example, $P_{final} = 0.1$ indicates that the auction was bought at the price 10% higher than μ_{ij}. $P_{buyout} = -0.1$ indicates that the buyout price was set to the price 10% lower than μ_{ij}. When $P_{start} = P_{buyout}$, the auction is sold at the fixed-price. When an auction with a buyout price was purchased at the buyout price, the equation $P_{final} = P_{buyout}$ is satisfied. The lower the price is, the value of the index is closer to -1. When $\mu_{ij} = 5000$ and $x = 1$, $P_{start} = -0.999$.

3.2 Data

The auction data of 50 items[1)] for 12 weeks[2)] in Yahoo! JAPAN auction was used. 11,921 auction data were examined. These data do not include auctions having no bid. The term in the defined indexes was set to two weeks and the data were divided into six terms[3)].

The auction data was extracted as follows. First, the auctions in the particular category whose titles match with the keyword were extracted. For example, if the intended item has his name "ABC123", the keyword is set to the name. Auctions whose titles include the keyword "ABC123" were extracted. Next, the unwanted data were removed by doing visual inspections. For example, the following auctions were removed: (1) auctions selling only the other items, (2) auctions selling the other items in addition to the intended item.

3.3 Buyout Option in Yahoo! JAPAN Auction

This section describes the buyout option in Yahoo! JAPAN auction. Yahoo! JAPAN provides a permanent buyout option that a fixed-price sale is allowed within an auction, while eBay provides a temporary buyout option called "buy-it-now." The option is called as "*Sokketsu Kakaku*" in Japanese. A seller can set a buyout price in addition to a start price. Setting a buyout price at the price equal to the start price corresponds to fixed-price selling. Once a buyout price is set, it cannot be changed until the end of the auction. While an auction is held, the start price and buyout price are disclosed. Even buyers bid at the price less than a buyout price in the auction, the buyout price is valid. In other words, if an auction with a buyout price is held, a buyer can quickly purchase the good by bidding at the buyout price.

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1) 50 items include the following item types: portable music player, laptop, Blu-ray Disc recorder, TV, digital camera, electronic dictionary, ETC, CD, DVD, game software, game console, Comic, Novel, old coin, trading card, gift certificate and health care item. Multiple items were selected from most item types.

2) March 9th 2009 – May 31st 2009

3) Six terms were defined as follows. Term 1: March 9th–22nd, Term 2: March 23rd–April 5th, Term 3: April 6th–19th, Term 4: April 20th–May 3rd, Term 5: May 4th–17th, Term 6: May 18th–31st.

Table 1: Classification of the data

Class	Frequency	Relative Frequency
No Buyout Price	5,348	0.449
$P_{start} < P_{buyout}$, "Auction"	1,161	0.097
$P_{start} < P_{buyout}$, "Buyout price"	3,734	0.313
$P_{start} = P_{buyout}$	1,678	0.141

Buyers can bid an auction at the price higher than the start price and less than the buyout price without bidding at the buyout price. However, if a buyer bid at the buyout price, the auction quickly finishes. Thus, an auction with a buyout price finishes at the price less than or equal to the buyout price. On the other hand, it is possible for a seller to improve his revenue by setting adequately a start price and a buyout price.

3.4 Result of Data Analysis

This section shows the result of data analysis.

Table 1 shows the result of classifying the data into four classes according to whether the auction had a buyout price and whether the auction was purchased at the buyout price. The classes in this table correspond to the following auctions. The class No Buyout Price includes the auctions where buyout prices were not set. The class $P_{start} < P_{buyout}$, "Auction" includes auctions where buyout prices were set and have no bid at the buyout prices. The class $P_{start} < P_{buyout}$, "Buyout price" includes the auctions where buyout prices were set and purchased at the buyout prices. The class $P_{start} = P_{buyout}$ includes the auctions where buyout prices set at the price equal to the start prices.

About 55% of the all data corresponds to auctions with buyout prices and about 45% of the data corresponds auctions purchased at buyout prices. In the auctions satisfying $P_{start} < P_{buyout}$, the number of the auctions purchased at buyout prices is about three times as many as the number of the auctions bought at the price less than buyout prices.

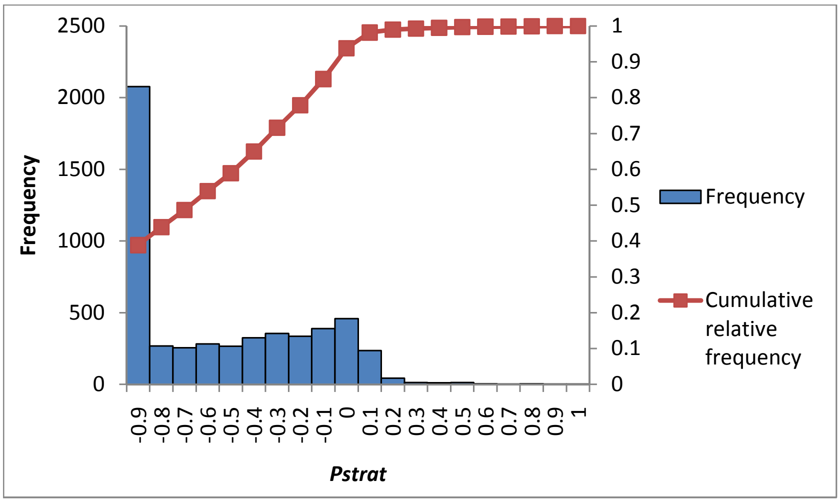

Figure 1: Distribution of the index of start price P_{start} (auctions without buyout prices)

3.4.1 Auctions without Buyout Prices

The actual data of auctions without buyout prices were analyzed. In the auctions, the sellers set only start prices. Figure 1 shows the distribution of the index of start price P_{start}. The highest relative frequency is 0.39 in the lowest class $-1 < P_{start} \leq -0.9$. In the other classes, the relative frequency in each class satisfying $P_{start} \leq 0$ is within the values from 5 to 7%. On the other hand, the relative frequency of auctions satisfying $P_{start} > 0$ is only 6.3% in auctions without buyout prices. Therefore, it is indicated that the start prices are set to the quite low price in many of the auctions without buyout prices.

3.4.2 Auctions with Buyout Prices

The actual data of auctions with buyout prices were analyzed. When a seller uses a buyout option, he must set both start price and buyout price. Therefore, the author investigated the setting of the combination of start price and buyout price in the auctions.

Figure 2 shows the distribution of the index of start price P_{start} in auctions with buyout prices. In the figure, many of the auctions with buyout prices are included in the class $-0.1 < P_{start} \leq 0$ and the class $0 < P_{start} \leq 0.1$. The class $-0.1 < P_{start} \leq 0$ has the highest relative frequency 0.27, and the second highest relative frequency is 0.24 in the class $0 < P_{start} \leq 0.1$. Therefore, on

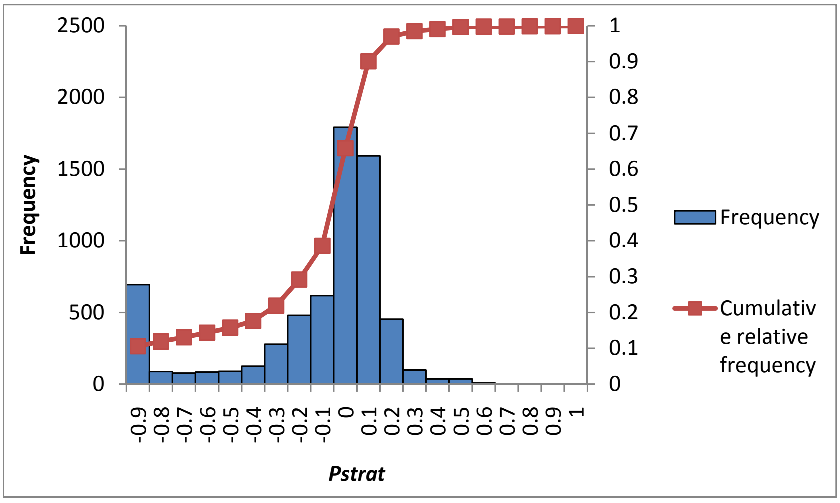

Figure 2: Distribution of the index of start price P_{start} (auctions with buyout prices)

the interval $-0.1 < P_{start} \leq 0.1$, the majority (51.5%) of auctions with buyout price are included. This result indicates that start prices are set to the prices near to the averages of final prices in many of auctions with buyout prices.

Figure 3 shows the distribution of the index of buyout price P_{buyout} in auctions with buyout prices. The class $0 < P_{buyout} \leq 0.1$ has the highest relative frequency 0.354, and the second highest relative frequency is 0.273 in the class $-0.1 < P_{buyout} \leq 0$. Therefore, on the interval $-0.1 < P_{start} \leq 0.1$, 62% of the auctions with buyout prices are included. This result indicates that buyout prices are set to the price near to the average of final prices in many of auctions with buyout prices.

Table 2 shows the result of examining the difference between the index of buyout price P_{buyout} and the index of start price P_{start}. The difference is defined as $P_{(b-s)} = P_{buyout} - P_{start}$. In the table, the auctions satisfying $P_{(b-s)} \leq 0.01$ accounts for 56.8%. For example, when $\mu_{ij} = 1000$ and $P_{(b-s)} = 0.01$, the amount of the difference between buyout price and start price is 10. Therefore, such auctions are regarded as fixed-price selling. This result indicates the majority of auctions with buyout prices are regarded as fixed-price selling at buyout prices.

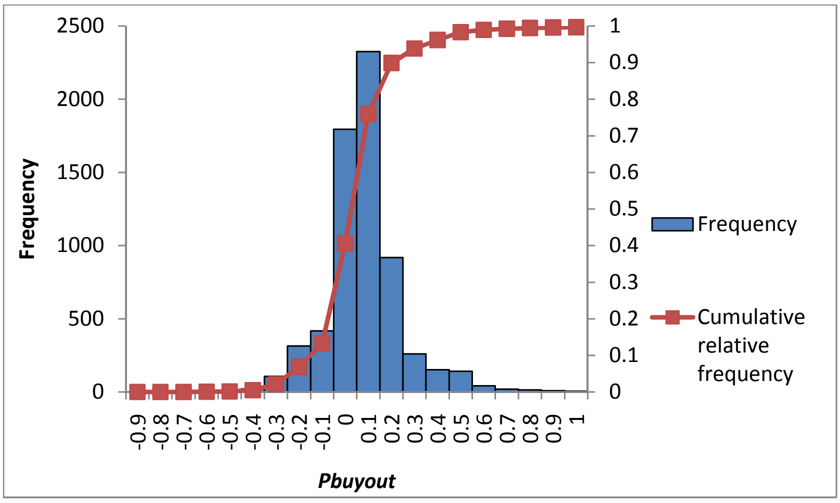

Figure 3: Distribution of the index of buyout price P_{buyout} (auctions with buyout prices)

Table 2: Distribution of $P_{(b-s)}$, the difference between the index of buyout price P_{buyout} and the index of start price P_{start} (auctions with buyout prices)

Class	Frequency	Relative Frequency
$0 = P_{(b-s)}$	1,678	0.255
$P_{(b-s)} \leq 0.01$	3,732	0.568

3.5 Typical Strategies of Sellers

The result of the data analysis indicates that the auctions corresponding to the following two types account for about half of the all data. The two strategies are regarded as the major strategies of sellers in Internet auctions.

TYPE 1: Auctions where buyout prices are not set and quite low start prices are set

TYPE 1 corresponds to auctions where buyout prices are not set and quite low start prices are set, i.e., $-1 < P_{start} \leq -0.9$. Since the start price is quite low, buyers easily bid the auction. However, the final price involves uncertainty. 40% of auctions without buyout prices can be classified into TYPE 1. They accounts for about 17% of the all data.

TYPE 2: Auctions where buyout prices are set at the price almost equal to the start prices

Table 3: Comparison of the index of final price P_{final} between typical sellers: TYPE 1 and TYPE 2

	TYPE 1	TYPE 2
Average of P_{final}	-0.008	-0.012
Standard deviation of P_{final}	0.171	0.141

TYPE 2 corresponds to auctions where buyout prices are set at the price almost equal to the start prices, i.e., $0 \leq (P_{buyout} - P_{start}) < 0.01$. They are regarded as fixed-price selling. If a buyer bid the auction, the final price is almost equal to the buyout price. 57% of auctions with buyout prices can be classified into TYPE 2. They accounts for about 31% of the all data.

Table 3 shows the comparison of the final prices between the two types. This table indicates that the standard deviation of P_{final} of TYPE 2 is lower than that of TYPE 1. However, the average of P_{final} of TYPE 1 is almost equal to that of TYPE 2.

Chapter 4 Model

Based on the above analysis, the author further investigates sellers' behaviors by building a model. The previous studies about buyout-price ascending auctions have discussed the seller's strategy whether using the buyout option or not. The model of Hindvegi et al. elucidates the conditions when the seller should use a buyout option and how to calculate the optimal buyout price [7]. However, the actual data shows that the buyout option is used in 55 % of auctions, while the buyout option is not used in 45% of auctions. The previous studies are difficult to explain this coexistence of TYPE 1 sellers and TYPE 2 sellers. To explain this situation, we have developed a model including two sellers and three buyers as follows.

In this chapter, the seller's strategy is restricted to the two strategies TYPE 1 and TYPE 2 defined earlier. Therefore, the strategy of a seller is selected from the following two strategies.

- Ascending auction by setting the lowest start price without a buyout price
- Fixed-price selling by setting a buyout price equal to the start price

4.1 Two-Stage Game where Two Sellers and Three Buyers Exist

The model dealing two-stage game where two sellers and three buyers exist is build.

4.1.1 Assumptions

Assumptions in the model are defined as follows.

In the model, two sellers and three buyers exist. The valuations of the buyers are drawn from the distribution function of F (the probability density function of f) on the interval $[\underline{v}, \overline{v}]$. All buyers are classified into risk-neutral or risk-averse. A constant probability q that a buyer is risk-averse is given. A risk-neutral buyer has a quasilinear utility function. A risk-averse buyer has a utility function $u_A(x)$. $u_A(x)$ is strictly convex Bernoulli utility function which is a continuous function such that $u_A(0) = 0$, $u_A^{'}(x) > 0$ and $u_A^{''}(x) < 0$. On the other hand, a seller has a quasilinear utility function and the valuation of the seller to its good is 0. The sellers can obtain the positive benefit and utility by selling their goods at any prices.

Suppose that two-stage game where two sellers S_1 and S_2 arrive sequentially. In stage 1, if seller S_1 provides a buyout price of B and there is at least one risk-averse buyer whose valuation is larger than or equal to B, the buyer purchases the good at the buyout price. In stage 2, seller S_2 prefers to sell his good in an ascending auction. In stage 1, if seller S_1 cannot sell his good at the buyout price, seller S_1 sells it at the same price setting in the stage 2.

4.2 Optimum Buyout Price

Consider the following four cases where how many buyers have their valuations larger than B.

(i) All three buyers have their valuations less than B

The probability of this case is $F(B)^3$. Since no buyer has his valuation larger than or equal to B, seller S_1 cannot sell the good at buyout price B. Therefore, the revenue of S_1 by selling at a buyout price in this case is 0.

(ii) A buyer has his valuation larger than or equal to B

The probability of this case is $3F(B)^2(1 - F(B))$. Even if a buyer has his valuation larger than or equal to B, seller S_1 cannot sell the good when the buyer is not risk-averse. Here, the probability that a buyer is risk-averse is given as q. Therefore, the revenue of S_1 by selling at a buyout price in this case is qB.

(iii) Two buyers have their valuations larger than or equal to B

The probability of this case is $3F(B)(1-F(B))^2$. If at least one of the two buyers is risk-averse, the auction of seller S_1 is purchased at a buyout price in stage 1. Even if the two buyers is risk-neutral, the auction of seller S_1 is bid at buyout price B after the auction of seller S_2 ascended to the price B in stage 2. Therefore, the revenue of S_1 by selling at a buyout price in this case is B.

(iv) All three buyers have their valuations larger than or equal to B

The probability of this case is $(1 - F(B))^3$. If at least one of the three buyers is risk-averse, the auction of seller S_1 is purchased at a buyout price in stage 1. Even if the three buyers are risk neutral, the auction of seller S_1 is bid at a buyout price B after the auction of seller S_2 ascended to the price B in stage 2. Therefore, the revenue of S_1 by selling at a buyout price in this case is B.

The expected revenue r_B can be obtained by summing up the expected revenue of each case from (i) to (iv). r_B is shown as

$$\begin{aligned} r_B &= 3F(B)^2(1-F(B))qB + 3F(B)(1-F(B))^2B \\ &\quad +(1-F(B))^3B. \end{aligned} \tag{1}$$

An optimal B of B^* can be obtained by solving the first-order condition of Eq.(1).

On the other hand, consider the expected revenues in the case where seller S_1 does not set a buyout price. The expected revenue r of the seller is shown as

$$r = \int_{\underline{v}}^{\overline{v}} 3yf(y)(1-F(y))^2 dy. \tag{2}$$

When seller S_1 does not set a buyout price, the two ascending auctions are held in stage 2. As a result, the final prices of the auctions are equal to the lowest valuation of all three buyers.

If buyout price B satisfies $r_B \geq r$, seller S_1 can improve his revenue by selling at a buyout price.

4.3 Evaluation of Model

This section shows the result of experiments using the proposal model.

4.3.1 Results of Experiments

a) Buyers' valuations depend on uniform distribution

First, the experiments were conducted assuming buyers' valuations depend on the uniform distribution. The uniform distribution function of F on $[\underline{v}, \overline{v}]$ is shown as

$$F(v) = \frac{v - \underline{v}}{\overline{v} - \underline{v}}.$$

The probability density function of f is shown as

$$f(v) = \frac{1}{\overline{v} - \underline{v}}.$$

The interval of F was set to $[100, 200]$ in the experiments. The expected revenue of seller S_1 setting buyout price B at the price on $[100, 200]$ was calculated by using Eq.(1) under the constant probability q. Additionally, the expected revenue in the case where seller S_1 does not set a buyout price was calculated by using Eq.(2).

Figure 4 shows the result of the experiment about the expected revenue in the case where seller S_1 sets buyout price B. This figure indicates the following things. First, the expected revenue is 100 when the buyout price is set to 100 the least valuation of buyers. Since all buyers have their valuations larger than or equal to 100, the auction with the buyout price equal to 100 must be bought by a buyer. The larger the buyout price is, the expected revenue increases in $B \leq B^*$. On the other hand, the larger the buyout price is, the expected revenue decreases in $B > B^*$. Buyout price B^* to maximize the expected revenue differs from the value of q. The value of B^* and r_{B^*} the revenue in the case where the buyout price is set is as follows. When $q = 0.1$, $B^* = 117$ and $r_{B^*} = 108.8$. When $q = 0.5$, $B^* = 127$ and $r_{B^*} = 114.4$. When $q = 0.9$, $B^* = 142$ and $r_{B^*} = 127.1$.

Figure 5 shows the comparison of the expected revenue between the case where seller S_1 sets a buyout price and the case where he does not set it. The expected revenue of seller S_1 in the case where he does not set a buyout price does not be effected by the value of q. It is calculated as $r = 125.0$ by using Eq.(2). On the other hand, the larger the value of q is, the expected revenue r_{B^*} in the case where seller S_1 sets B^* increases. In the figure, when $q \geq 0.86$, the condition $r_{B^*} \geq r$ is satisfied and the expected revenue is improved by setting the buyout prices.

b) Buyers' valuations depend on the exponential distribution

In the experiments of a), the case where the valuations of the buyers depend on the uniform distribution was considered. In the real auctions, however, if the price of the good is lower, the more buyers who desire to purchase it at

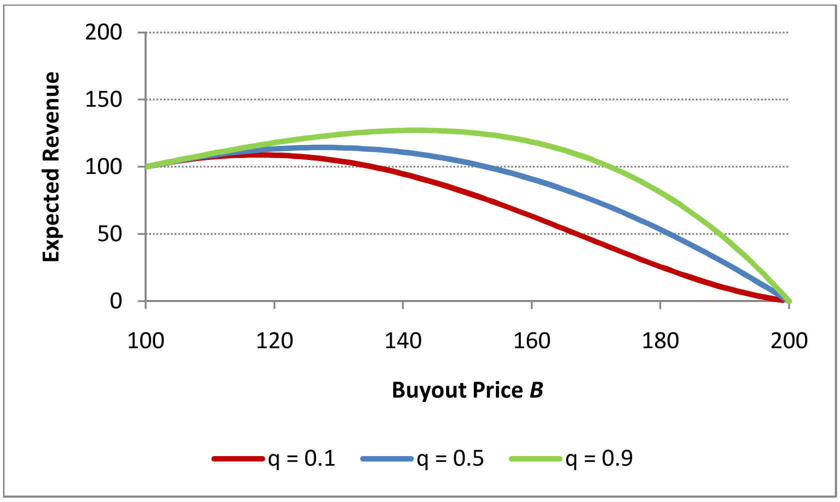

Figure 4: Expected revenue of seller S_1 in the case where he sets buyout price B (F: uniform distribution on [100,200])

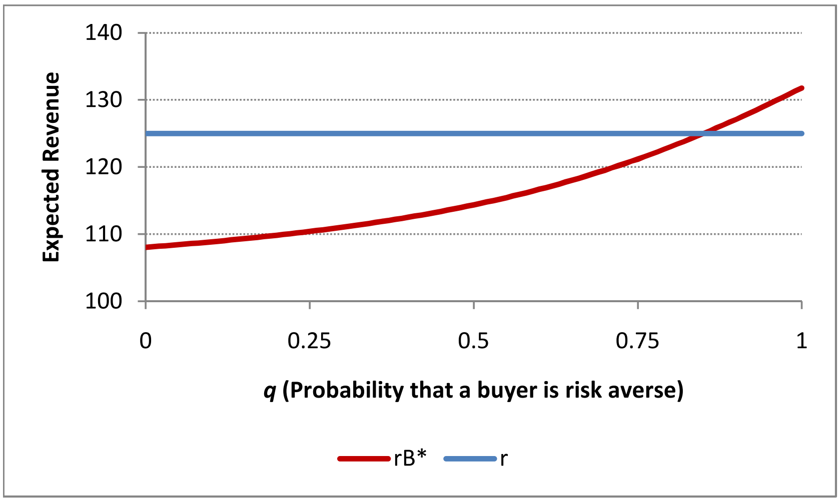

Figure 5: Relation between q and expected revenue of seller S_1 (F: uniform distribution on [100,200])

the price may exist. In order to discuss the situation, consider the case where the valuations of the buyers depends on the exponential distribution F on the interval $[\underline{v}, \overline{v}]$. F is written as

$$F(v) = k_1 - k_2 \cdot \exp\left(-\frac{v - \underline{v}}{\overline{v} - \underline{v}}\right)$$

where k_1 and k_2 satisfy $F(\underline{v}) = 0$ and $F(\overline{v}) = 1$. They are calculated as

$$k_1 = k_2 = \frac{e}{e - 1}.$$

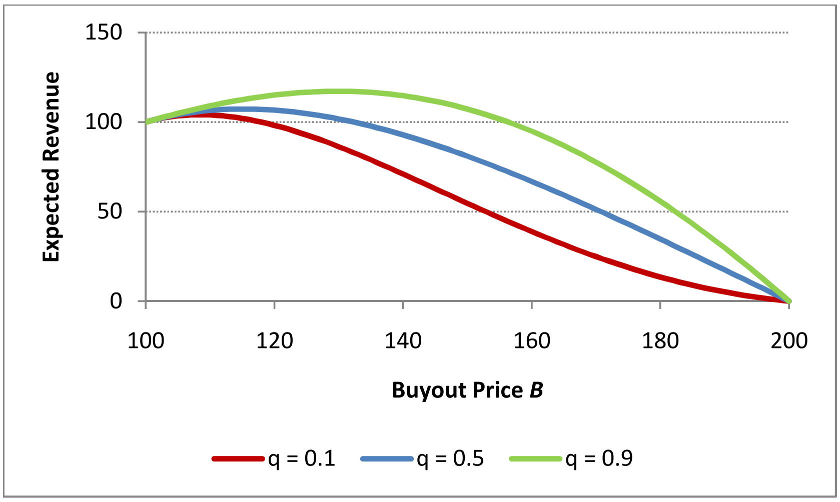

Figure 6: Expected revenue of seller S_1 in the case where he set buyout price B (F: exponential distribution on [100,200])

The probability density function of f is expressed as

$$f(v) = \frac{k_2}{\overline{v} - \underline{v}} \cdot \exp\left(-\frac{v - \underline{v}}{\overline{v} - \underline{v}}\right).$$

In this experiments, the interval of F was set to $[100, 200]$.

Figure 6 shows the result of the experiment about the expected revenue in the case where seller S_1 sets buyout price B. This figure indicates the following things. First, as well as the case of the uniform distribution, the expected revenue is 100 in the case where the buyout price is set at 100 equal to minimum valuation of buyers. The larger the buyout price is, the more expected revenue is obtained in $B \leq B^*$. On the other hand, the larger the buyout price is, the less expected revenue is obtained in $B > B^*$. Buyout price B^* to maximize the expected revenue differs from the value of q. These results are same as the case of the uniform distribution. The values of B^* and r_{B^*} are calculated as follows. In the case $q = 0.1$, $B^* = 108$ and $r_{B^*} = 104.0$. In the case $q = 0.5$, $B^* = 115$ and $r_{B^*} = 107.2$. In the case $q = 0.9$, $B^* = 130$ and $r_{B^*} = 117.2$. In all the value of q, buyout price B^* and expected revenue r_{B^*} in the case of exponential distribution are less than the case of uniform distribution.

Figure 7 shows the comparison of the expected revenue between the case where seller S_1 sets a buyout price and the case where he does not set it. The expected revenue of seller S_1 in the case he does not set a buyout price does

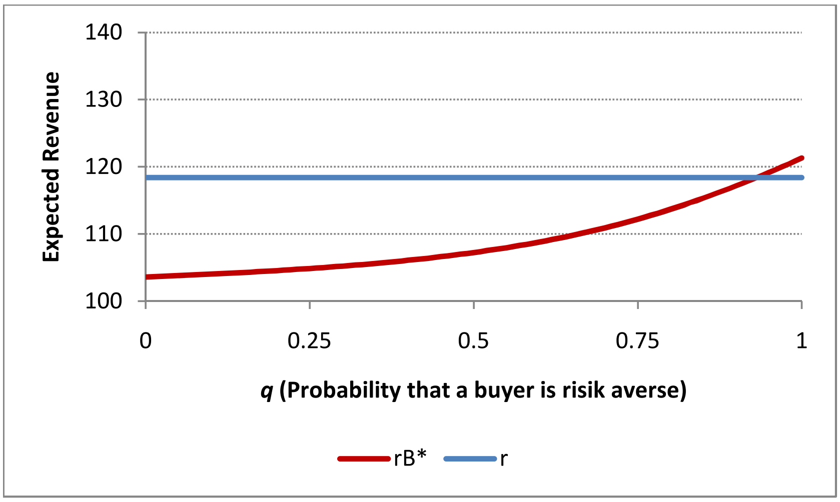

Figure 7: Relation between q and expected revenue of seller S_1 (F: exponential distribution on [100,200])

not be effected by the value of q. It is calculated as $r = 118.4$ by using Eq.(2). On the other hand, the higher the value of q is, the more expected revenue r_{B^*} is obtained in each the value q. In the figure, when $q \geq 0.94$, the condition $r_{B^*} \geq r$ is satisfied and the expected revenue is improved by setting the buyout prices. Compared to the case of the uniform distribution, the larger value of q is required to obtain the more revenue than the revenue without a buyout price.

These results indicate the following things. Under the same interval of F, compared to the case of the uniform distribution, the larger value of q is required to improve the revenue and the increase of the revenue is less in the case of the exponential distribution.

c) Relation between distribution of buyers' valuations and buyout price

The relation between the width of the interval of distribution F and buyout price are examined.

The method of experiments is shown as follows. The median value in the interval of F is defined as τ. The width of interval of F is defined as $d = \overline{v} - \underline{v}$. In this case, the valuations of buyers depend on the distribution on $[\tau - d/2, \tau + d/2]$. F was set to the uniform distribution and $\tau = 150$. Since q was set to 1, r_{B^*} in the following experiments is the maximum expected revenue obtained in the

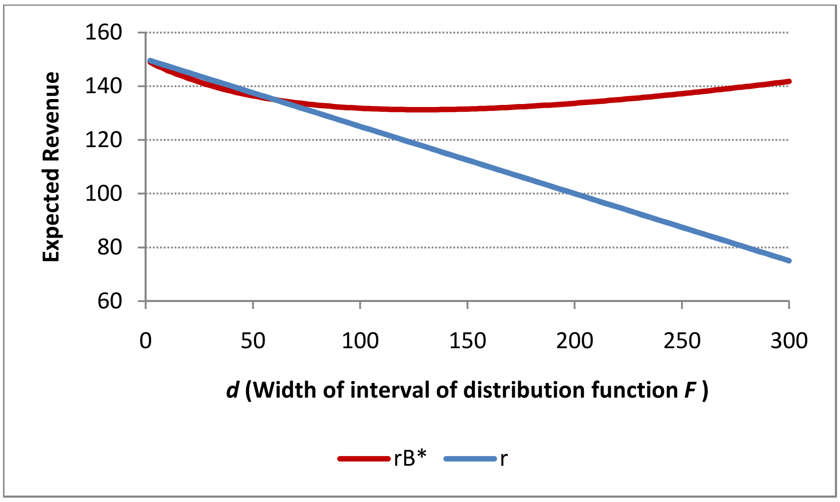

Figure 8: Relation between the width of interval of F and seller S_1's expected revenue ($q = 1$, F: uniform distribution on $[150 - d/2, 150 + d/2]$)

condition.

Figure 8 shows the relation between d and seller S_1's expected revenue when d was increased by two from $d = 2$ (F on $[149, 151]$) to $d = 300$ (F on $[0, 300]$.) The revenue r in the case where seller S_1 does not set a buyout price decreases monotonically as the interval increases. This is because the final prices of the auctions without buyout prices depend on the lowest valuation of the buyers. On the other hand, the revenue r_{B^*} in the case where buyout price B^* was set increases when $d > 62$ (F on $[119, 181]$) . The wider the interval of F is, the more buyers have large valuations. Therefore, if the seller sets a large buyout price, a buyer who has the larger valuation purchases the good at the buyout price.

Finally, the increase of the revenue by selling the good at the buyout price $r_{B^*} - r$ was analyzed. Figure 9 shows the relation between d and $r_{B^*} - r$ in $\tau = 150$. When d is higher than a threshold value, $r_{B^*} - r$ is positive. In the case of the uniform distribution, the condition is $d \geq 62$. In the case of the exponential distribution, it is $d \geq 110$. When these conditions are satisfied, $r_{B^*} - r > 0$ is satisfied and $r_{B^*} - r$ increases monotonically as the interval increases.

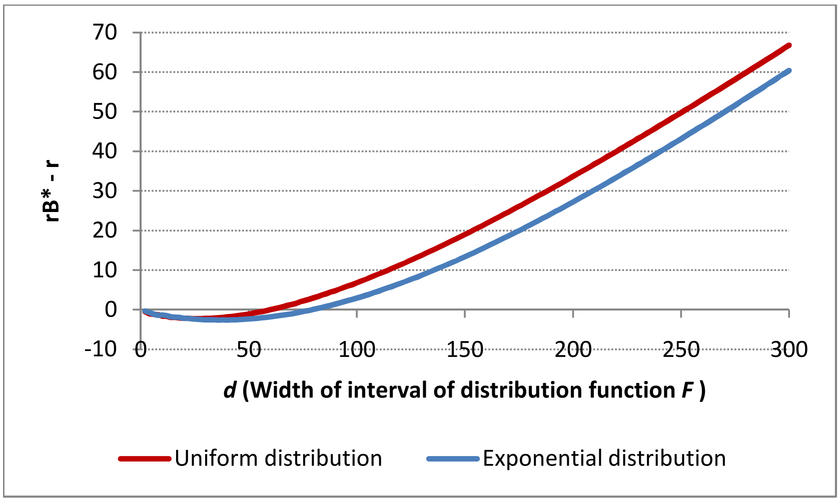

Figure 9: Relation between the width of interval of F and increase of seller S_1's expected revenue $r_{B^*} - r$ ($q = 1$, F: distribution function on $[150 - d/2, 150 + d/2]$)

4.3.2 Comparison of Seller's Revenue

The author has carried out simulation to examine the sellers' revenues.

F was set to the uniform distribution on $[100, 200]$. By increasing q from 0 to 1 by 0.01, B^* in each value of q was obtained. For each q, $100,000$ examples were created. The averages of the revenues in the case where seller S_1 sets B^* and the case where seller S_1 does not set a buyout price were calculated.

Figure 10 shows the comparison of each seller's revenue. When no seller sets a buyout price ("No Buyout Price" in the figure), the revenue of seller S_1 is equal to the revenue of seller S_2. The revenue of seller S_1 who sets buyout price B^* increases as q increases. In the experiment, when $q \geq 0.85$, the revenue of seller S_1 with buyout price B^* is higher than one without buyout price. On the other hand, the revenue of seller S_2 in the case where seller S_1 sets the buyout price is always larger than the case without buyout price S_1. In addition, as the value of q increases, the revenue of seller S_2 decreases monotonically. In this experiment, when $q \geq 0.91$, the revenue of seller S_1 who set B^* is larger than the revenue of seller S_2.

Figure 11 shows the comparison of total revenue of two sellers. In the figure, when $q \geq 0.78$, the total revenue of two sellers with buyout price B^* is larger

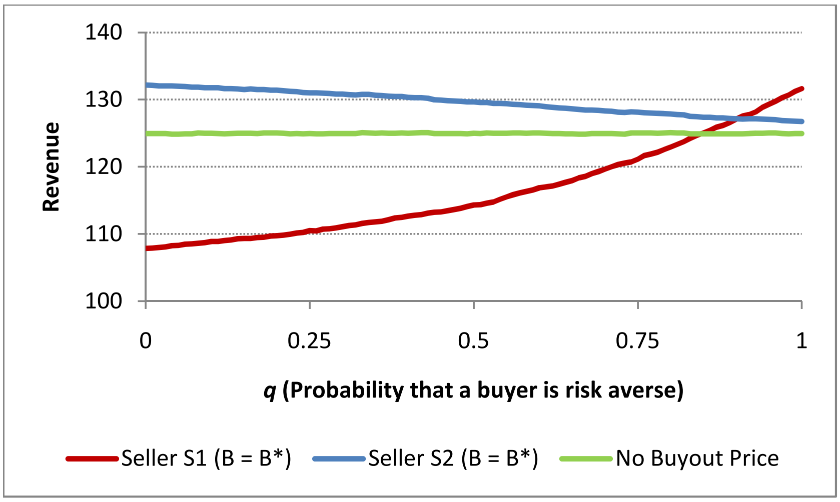

Figure 10: Comparison of a seller's revenue (F: uniform distribution on [100,200])

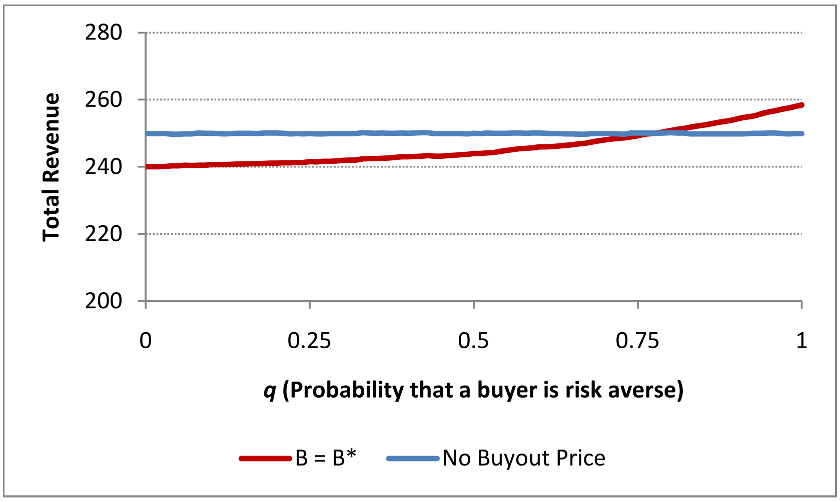

Figure 11: Comparison of total revenue of two sellers (F: uniform distribution on [100,200])

than the total revenue without buyout price.

An interesting result from the experiment is that, when seller S_1 sells at a buyout price, seller S_2 can obtain the larger revenue than the revenue in the case where seller S_1 does not set it. The higher the value of q is, the higher the revenue of seller S_1 is and the lower the revenue of seller S_2 is. However, the revenue of S_2 is always improved by S_1's fixed-price selling at a buyout price.

4.3.3 Consideration

The result of the experiments using the model shows the following things.

First, when seller S_1 sets buyout price B^* to maximize his expected revenue, whether his expected revenue is larger than the revenue without a buyout price depends on the following two things. First, the larger probability q is, the larger expected revenue r_{B^*} is obtained. If there is no risk-averse buyers ($q = 0$), the expected revenue does not be improved by selling at a buyout price. Secondly, the expected revenue depends on $d = \overline{v} - \underline{v}$, the width of interval of the distribution the buyers' valuations depend on. When the median of the interval $(\overline{v} + \underline{v})/2$ is the same value, the expected revenue in ascending auctions without buyout prices decreases as the interval gets wide. On the other hand, if d is larger than the threshold value, the expected revenue with buyout price increases as the wider the interval is. In other words, the larger the width of the interval of the distribution is, the revenue of the seller can be enhanced by setting the buyout price.

In addition, the simulation experiment results indicate the following point. When seller S_1 selects fixed-price selling at a buyout price, the revenue of seller S_2 is larger than the revenue in the case where seller S_1 does not set a buyout price. Although seller S_2 does not change his behavior, his revenue is improved by seller S_1' selecting fixed-price selling at a buyout price.

Chapter 5 Extension of the Model

In Chapter 4, seller's strategy is restricted to the following two major strategies obtained from the data analysis: "ascending auction from the lowest start price" and "fixed-price selling at a buyout price". However, in the actual Internet auctions, some sellers select the other strategies. When strategy of a seller setting a buyout price is limited to fixed-price selling, the seller cannot always benefit by using the buyout option for his good. If the seller can set an appropriate combination of start price and buyout price, it is possible for the seller to always benefit by using the buyout option. This chapter extends seller's strategy and discusses the price setting in the perfect Bayesian Nash equilibrium.

In addition to it, in Chapter 4, both valuations of seller S_1 and S_2 are set to 0. However, considering the actual situation, some sellers have their high valuations to their own goods. In this case, the sellers attempt to avoid selling at the price less than their valuations. Therefore, they need to set start prices at the price larger than or equal to their valuations. Thus, this chapter also discuss the situation where the sellers have their valuation larger than 0 and the value of start prices they can set are limited according to their valuations.

5.1 Valuations of Sellers

Suppose that a seller does not prefer to sell his good at the price less than his valuation. Therefore, the seller attempts to maximize his expected revenue under the condition that he sells his good at the price larger than or equal to his valuation.

According to sellers' valuations, sellers are classified into the following two types. The first type is the seller who has his valuation $v_{(1)}$ satisfying $0 \leq v_{(1)} < \underline{v}$. Since he has his valuation less than $\underline{v}$ the least valuation of buyers, he can set any price on the interval $[\underline{v}, \overline{v}]$. The second type is the seller who has his valuation $v_{(2)}$ satisfying $\underline{v} \leq v_{(2)} \leq \overline{v}$. Since he has his valuation larger than or equal to $\underline{v}$, he can only set any price on the interval $v_{(2)} \leq B \leq \overline{v}$. In this case, the bound

$$0 \leq v_{(1)} < \underline{v} \leq v_{(2)} \leq \overline{v} \tag{3}$$

is satisfied. In this paper, the seller who has his valuation larger than $\overline{v}$ is not considered, because he cannot sell the good at the price larger than or equal to his valuation.

5.2 Assumptions

Assumptions in the extended model are described.

As well as Chapter 4, the extended model discusses the situation where two sellers and three buyers exist. Suppose that two-stage game where two sellers S_1 and S_2 arrive sequentially. First, seller S_1 sets start price p_{s1} and buyout price B. The prices are disclosed. Seller S_2 sets start price p_{s2} considering them. In stage 1, if seller S_1 provides buyout price B and there is at least one risk-averse buyer whose valuation is larger than or equal to B, the buyer purchases the good of S_1 at the buyout price. In stage 2, seller S_2 prefers to sell his good in an ascending auction.

The valuations of the buyers are drawn from the distribution function of F (the probability density function of f) on the interval $[\underline{v}, \overline{v}]$. All buyers are classified into risk-neutral or risk-averse. The constant probability q that a buyer is risk-averse is given. A risk-neutral buyer has a quasilinear utility function. A risk-averse buyer has a utility function $u_A(x)$ as well as Chapter 4. On the other hand, the valuation of seller S_1 to his good is v_{s1} and the valuation of seller S_1 to his good is v_{s2}. If a seller sells his good at the price less than his valuation, the seller has an enormous negative utility. Therefore, the seller must set start price and buyout price at the prices larger than or equal to his valuation. Assume that the utility is equal to the revenue of the auction when the seller sets the price larger than or equal to his valuation.

Finally, define the following equations to use discussion about the expected revenues of sellers. When the valuations of the buyers are drawn from the distribution function of F (the probability density function of f) on the interval $[\underline{v}, \overline{v}]$, define $v_{[\alpha,\beta]}(n, m)$ as the m-th largest valuation of the n buyers who have their valuations larger than or equal to α and less than or equal to β. The conditions $\underline{v} \leq \alpha$ and $\beta \leq \overline{v}$ are satisfied. $v_{[\alpha,\beta]}(n,m)$ is shown as

$$v_{[\alpha,\beta]}(n,m) = \int_{\alpha}^{\beta} \binom{n}{1}\binom{n-1}{m-1} f_{[\alpha,\beta]}(z)(1-F_{[\alpha,\beta]}(z))^{m-1}(F_{[\alpha,\beta]}(z))^{n-m} z dz \quad (4)$$

where $f_{[\alpha,\beta]}(x)$ and $F_{[\alpha,\beta]}(x)$ are expressed as follows:

$$f_{[\alpha,\beta]}(x) = \frac{f(x)}{F(\beta)-F(\alpha)}, \quad (5)$$

$$F_{[\alpha,\beta]}(x) = \frac{F(x)-F(\alpha)}{F(\beta)-F(\alpha)}. \quad (6)$$

The following equations hold:

$$F_{[\alpha,\beta]}(x) = \int_{\alpha}^{x} f_{[\alpha,\beta]}(x)dx \ (\alpha \leq x \leq \beta),$$
$$F_{[\alpha,\beta]}(\alpha) = 0 \ , \ F_{[\alpha,\beta]}(\beta) = 1.$$

5.3 Strategies of Sellers

Each seller selects a different strategy according to the order of arrival at the market.

First, consider the strategy of the first seller S_1. Since seller S_1 arrives at Stage 1, his revenue may be improved by setting buyout price B. Therefore, seller S_1 attempts to set start price p_{s1} and buyout price B. When seller S_1 has his valuation v_{s1}, p_{s1} and B must satisfy $v_{s1} \leq p_{s1} \leq B$.

Secondly, consider the strategy of the second seller S_2. Seller S_2 arrives at stage 2. From the assumption, risk-averse buyers promptly purchase the good at a buyout price in only stage 1. Since seller S_2 starts to sell in stage 2, he cannot improve his revenue by setting a buyout price. Therefore, seller S_2 sets only start price p_{s2} and does not set a buyout price. When seller S_2 has his valuation v_{s2}, p_{s2} must satisfy $v_{s2} \leq p_{s2}$.

Since seller S_1 sets the prices earlier than seller S_2, seller S_2 can respond to the price setting of S_1. Seller S_2 can select the start price p_{s2} satisfying $v_{s2} \leq p_{s2}$ in order to maximize his expected revenue. The start price maximizing his expected revenue r_{s2} is expressed as ${p_{s2}}^*$. On the other hand, seller S_1 select the strategy considering the response of seller S_2: setting start price$p_{s2}{}^*$. Seller S_1 selects the combination of start price p_{s1} and buyout price B satisfying $v_{s1} \leq p_{s1} \leq B$ in order to maximize his expected revenue r_{s1}. The start price and buyout price maximizing his expected revenue r_{s1} are expressed as $p_{s1}{}^*$ and B^*.

5.4 The Case where No Seller Sets a Buyout Price

This section describes the expected revenues of the sellers in the case where no seller sets a buyout price. The equations in this section can be used to calculate the expected revenues of the sellers in the case where seller S_1 sets buyout price. This case is divided into the following two cases: a) two sellers set unequal start prices and b) two sellers set the same start price.

a) Two sellers set unequal start prices

First, consider the case where the two sellers set unequal start prices. When no seller sets a buyout price, two ascending auctions are held in stage 2. Therefore, two sellers do not be distinguished by the order of arrival. To identify the two start prices, the higher start price is called as p_s^H and the lower start price is called as p_s^L, where $p_s^H > p_s^L$ is satisfied. The seller providing p_s^H is called S^H and the valuation of the seller is v^H. On the other hand, the seller providing p_s^L is called S^L and the valuation of the seller is v^L. There are following four cases where how many buyers have their valuations larger than B.

(i) All three buyers have their valuations less than p_s^L

The probability of this case is $Prob_1 = F(p_s^L)^3$. In this case, no buyer can bid the auctions. Therefore, the revenue of S^H is 0 and the revenue of S^L is 0.

(ii) A buyer has his valuation larger than or equal to p_s^L

The probability of this case is $Prob_2 = 3(1 - F(p_s^L))F(p_s^L)^2$. In this case, no buyer can bid the auction of S^H. Since the only buyer whose valuation is larger than or equal to p_s^L can bid the auction of seller S^L, the price of the auction does not ascend. Therefore, the revenue of S^H is 0 and the revenue of S^L is p_s^L.

(iii) Two buyers have their valuation larger than or equal to p_s^L and no buyer of the two has his valuation larger than or equal to p_s^H

The probability of this case is $Prob_3 = 3(F(p_s^H) - F(p_s^L))^2 F(p_s^L)$. In this case, no buyer can bid the auction of seller S^H. On the hand, the auction of seller S^L ascends by the price equal to the lower valuation of the two buyers, who have their valuations larger than or equal to p_s^L. Therefore, the revenue of S^H is 0 and the revenue of S^L is $v_{[p_s^L, p_s^H]}(2, 2)$ calculated by using Eq.(4).

(iv) Two buyers have their valuations larger than or equal to p_s^L and a buyer of the two has his valuation larger than or equal to p_s^H

The probability of this case is $Prob_4 = 6(F(p_s^H) - F(p_s^L))(1 - F(p_s^H))F(p_s^L)$. The buyer who has his valuation larger than or equal to p_s^H can bid the auction of seller S^L at the price less than price p_s^H. Thus he does not bid the auction of seller S^H. The auction of seller S^L ascends by the valuation of the buyer who has his valuation larger than or equal to p_s^L and less than p_s^H. Therefore, the revenue of S^H is 0 and the revenue of S^L is $v_{[p_s^L, p_s^H]}(1, 1)$ calculated by using Eq.(4).

(v) All three buyers have their valuations larger than or equal to p_s^L and no buyer of the three has his valuation larger than or equal to p_s^H

The probability of this case is $Prob_5 = (F(p_s^H) - F(p_s^L))^3$. No buyer can bid the auction of seller S^H. Since all three buyers have their valuation larger than or equal to p_s^L, the auction of seller S^L ascends by the second largest valuation of the three buyers. Therefore, the revenue of S^H is 0 and the revenue of S^L is $v_{[p_s^L, p_s^H]}(3, 2)$ calculated by using Eq.(4).

(vi) All three buyers have their valuations larger than or equal to p_s^L and a buyer of the three has his valuation larger than or equal to p_s^H

The probability of this case is $Prob_6 = 3(F(p_s^H) - F(p_s^L))^2(1 - F(p_s^H))$. Since the buyer who has his valuation larger than or equal to p_s^H can bid the auction of S^L at the price less than p_s^H, he does not bid the auction of S^H. The auction of seller S^L ascends by the largest valuation of the two buyers who have their valuations larger than or equal to p_s^L and less than

p_s^H. Therefore, the revenue of S^H is 0 and the revenue of S^L is $v_{[p_s^L, p_s^H]}(2,1)$ calculated by using Eq.(4).

(vii) Two buyers have their valuations larger than or equal to p_s^H
The probability of this case is $Prob_7 = 3(1 - F(p_s^H))^2 F(p_s^H)$. The two buyers can bid the auction of seller S^H and can also bid the auction of seller S^L. If the one buyer bids the auction of seller S^L at the price p_s^H, the other buyer bids the auction of seller S^H. Therefore, the revenue of S^H is p_s^H and the revenue of S^L is equally p_s^H.

(viii) All three buyers have their valuations larger than or equal to p_s^H
The probability of this case is $Prob_8 = (1 - F(p_s^H))^3$. In this case, all three buyers can bid the auction of seller S^H and the auction of seller S^L. Both auctions ascend by the third largest valuation of them. Therefore, the revenue of S^H is $v_{[p_s^H, \overline{v}]}(3,3)$ calculated by using Eq.(4) and the revenue of S^L is equally $v_{[p_s^H, \overline{v}]}(3,3)$.

From (i) to (viii), the expected revenue r^L of seller S^L and r^H of seller S^H in the case where the two sellers do not set buyout prices and set start price p_s^L and p_s^H ($p_s^L < p_s^H$) are expressed as follows:

$$\begin{aligned} r^L &= Prob_2\ p_s^L + Prob_3\ v_{[p_s^L, p_s^H]}(2,2) \\ &\quad + Prob_4\ v_{[p_s^L, p_s^H]}(1,1) + Prob_5\ v_{[p_s^L, p_s^H]}(3,2) \\ &\quad + Prob_6\ v_{[p_s^L, p_s^H]}(2,1) + Prob_7\ p_s^H + Prob_8\ v_{[p_s^H, \overline{v}]}(3,3), \end{aligned} \tag{7}$$

$$r^H = Prob_7\ p_s^H + Prob_8\ v_{[p_s^H, \overline{v}]}(3,3). \tag{8}$$

From Eq.(7) and (8), $r^L \geq r^H$ is always satisfied. Therefore, when the two sellers sell without buyout prices, the seller who sets the lower start price can obtain the larger expected revenue. In order to set the lower start price, the seller sets the start price larger than the valuation of the other seller. Therefore, seller S^L can obtain the expected revenue larger than seller S^H by setting the start price satisfying $v^L \leq p_s^L < v^H$.

b) Two sellers set the same start price

Consider the case where the two sellers set the same start price. The start price is called as p_s^M. In this case, both p_s^L and p_s^H in 5.4 a) correspond to p_s^M. The expected revenue in this case can be considered in much the same way. If a buyer has his valuation larger than or equal to p_s^M, the buyer randomly bid an auction. There are four cases where how many buyers have their valuations larger than p_s^M. Therefore, in the case where two sellers does not set buyout prices and set the same start price p_s^M, each seller's expected revenue can be expressed as

$$\begin{aligned} r^M &= 3F(p_s^M)^2(1-F(p_s^M))\ \frac{p_s^M}{2} + 3F(p_s^M)(1-F(p_s^M))^2\ p_s^M \\ &\quad +(1-F(p_s^M))^3\ v_{[p_s^M,\overline{v}]}(3,3). \end{aligned} \tag{9}$$

5.5 Strategy of the Seller who Sets a Buyout Price

This section discusses the strategy of seller S_1 in the case where seller S_1 sets a buyout price in addition to a start price.

5.5.1 Expected Revenue

Consider the expected revenue of seller S_1 in this case. Start price p_{s1} and buyout price B which seller S_1 set satisfy the bound $v_{s1} \leq p_s \leq B$. As discussion in 4.2, There are four cases where how many buyers have their valuations larger than B. When seller S_1 sets start price p_{s1} and buyout price B, the expected revenue r_{s1} is shown as

$$\begin{aligned} r_{s1} &= F(B)^3 r_{(0)} + 3F(B)^2(1-F(B))((1-q)r_{(1)} + qB) \\ &\quad +3F(B)(1-F(B))^2((1-q)^2 r_{(2)} + (1-(1-q)^2)B) \\ &\quad +(1-F(B))^3 B \end{aligned} \tag{10}$$

where $r_{(0)}$, $r_{(1)}$ and $r_{(2)}$ are the expected revenues in the cases where no buyer purchases the good of seller S_1 in stage 1. In these cases, no buyer purchases the good of seller S_1 at a buyout price in stage 1. $r_{(0)}$ is the expected revenue in the case where all three buyers have their valuations less than B. $r_{(1)}$ is the expected revenue in the case where a buyer has his valuation larger than or equal to B and no buyer purchases the good in stage 1. $r_{(2)}$ is the expected revenue in the case where two buyers have their valuations larger than or equal to B and no buyer purchases the good in stage 1.

Consider $r_{(0)}$, $r_{(1)}$ and $r_{(2)}$ in Eq.(10) by dividing two cases a) and b) according to the relation between p_{s2} and B.

a) The other seller's start price is less than or equal to the buyout price ($p_{s2} \leq B$)

Consider the case where seller S_2 sets start price p_{s2} less than or equal to buyout price B which seller S_1 sets ($p_{s2} \leq B$). This case is divided into a-1) and a-2) according to the relation between p_{s1} and p_{s2}.

a-1) Two sellers set unequal start prices ($p_{s1} < p_{s2}$ or $p_{s1} > p_{s2}$)

Consider $r_{(0)}$, $r_{(1)}$ and $r_{(2)}$ in Eq.(10) in the case where two sellers set unequal start prices. The start prices of two sellers are called as follows: the higher start price is called as p_s^H and the lower start price is called as p_s^L where $p_s^H > p_s^L$. The seller setting p_s^H is called S^H and the valuation of the seller is v^H. On the other hand, the seller setting the lower start price p_s^L is called S^L and the valuation of the seller is v^L.

First, $r_{(0)}$ is calculated converting f and F in Eqs.(7), (8) and (9) into $f_{[\underline{v},B]}$ by using Eq.(5) and $F_{[\underline{v},B]}$ by using Eq.(6).

Secondly, consider $r_{(1)}$ in this case. Since no buyer purchases the good at the price larger than or equal to the buyout price, two sellers do not be distinguished by the order of arrival. When a buyer has his valuation v larger than or equal to B, at least one buyer's valuation satisfy the condition $p_s^L < p_s^H \leq B \leq v$. Call such a buyer as $Buyer_{(B)}$. Consider the expected revenue in this case by dividing into the following five cases.

(i) A buyer has his valuation larger than or equal to p_s^L

The two buyers except $Buyer_{(B)}$ have their valuations less than p_s^H. The probability of this case is $Prob_9 = F_{[\underline{v},B]}(p_s^L)^2$. In this case, no buyer can bid the auction of S^H and the one buyer can bid the auction of S^L. Therefore, the revenue of S^H is 0 and the revenue of S^L is p_s^L.

(ii) Two buyers have their valuations larger than or equal to p_s^L, and the one of the two has valuation larger than or equal to p_s^H

The one of the two buyers except $Buyer_{(B)}$ has larger than or equal to p_s^L and less than p_s^H. The probability of this case is $Prob_{10} = 2F_{[\underline{v},B]}(p_s^L)(F_{[\underline{v},B]}(p_s^H) - F_{[\underline{v},B]}(p_s^L))$. In this case, the buyer who has highest valuation bid the auction of S^L without bidding the auction of S^H. The price in auction of S^L

ascends by the valuation of the buyer who has his valuation larger than or equal to p_s^L and less than p_s^H Therefore, the revenue of S^H is 0 and the revenue of S^L is $v_{[p_s^L,p_s^H]}(1,1)$ calculated by using Eq.(4).

(iii) All three buyers have their valuations larger than or equal to p_s^L, and the one of the three has valuation larger than or equal to p_s^H
The other two buyers except $Buyer_{(B)}$ valuation are larger than or equal to v^L and less than v^H. The probability of this case is $Prob_{11} = (F_{[\underline{v},B]}(p_s^H) - F_{[\underline{v},B]}(p_s^L))^2$. In this case, the buyer who has highest valuation bid the auction of S^L without bidding the auction of S^H. On the other hand, the price in auction of S^L ascends by the largest valuation of the two buyers who have valuations larger than or equal to p_s^L and less than p_s^H. Therefore, the revenue of S^H is 0 and the revenue of S^L is $v_{[p_s^L,p_s^H]}(2,1)$ calculated by using Eq.(4).

(iv) Two buyers have their valuations larger than or equal to p_s^H
The one of the two buyers except $Buyer_{(B)}$ has his valuation larger than or equal to p_s^H and the another one has his valuation less than p_s^H. The probability of this case is $Prob_{12} = 2(1 - F_{[\underline{v},B]}(p_s^H))(1 - F_{[\underline{v},B]}(p_s^H))$. The prices of the two auctions ascends by the p_s^H. Therefore, the revenue of S^H is $p_s^H$and the revenue of S^L is equally p_s^H.

(v) All three buyers have their valuations larger than or equal to p_s^H
The two buyers except $Buyer_{(B)}$ have their valuations larger than or equal to p_s^H. The probability of this case is $Prob_{13} = (1 - F_{[\underline{v},B]}(p_s^H))^2$. The prices of the two auctions ascends by the second largest valuation of the two buyers who have their valuations larger than or equal to p_s^H and less than B. Therefore, the revenue of S^H is $v_{[p_s^H,B]}(2,2)$ calculated by using Eq.(4) and the revenue of S^L is equally $v_{[p_s^H,B]}(2,2)$.

From (i) to (v), the expected revenue $r_{(1)}^L$ of seller S^L and $r_{(1)}^H$ of seller S^H in the case where two sellers set start price p_s^L and p_s^H are expressed as

$$\begin{aligned} r_{(1)}^L &= Prob_9\ p_s^L + Prob_{10}\ v_{[p_s^L,p_s^H]}(1,1) \\ &\quad + Prob_{11}\ v_{[p_s^L,p_s^H]}(2,1) + Prob_{12}\ p_s^H + Prob_{13}\ v_{[p_s^H,B]}(2,2), \end{aligned} \tag{11}$$

$$r_{(1)}^H = Prob_{12}\ p_s^H + Prob_{13}\ v_{[p_s^H,B]}(2,2). \tag{12}$$

Finally, $r_{(2)}$ can be calculated as follows. In this case, at least two buyers are considered as $Buyer_{(B)}$ whose valuation satisfies $p_s^L < p_s^H \leq B \leq v$. Consider the expected revenue in this case by dividing into the following two cases.

(i) Two buyers have their valuations larger than or equal to p_s^H

The buyer who is not $Buyer_{(B)}$ has his valuation less than p_s^H. The probability of this case is $F_{[\underline{v},B]}(p_s^H)$. The prices of the two auctions ascends by p_s^H. Therefore, the revenue of S^H is p_s^H and the revenue of S^L is equally p_s^H.

(ii) All three buyers have their valuations larger than or equal to p_s^H

The buyer who is not $Buyer_{(B)}$ has his valuation larger than or equal to p_s^H. The probability of this case is $(1 - F_{[\underline{v},B]}(p_s^H))$. The prices of the two auctions ascends by the valuation of the buyer who has his valuation larger than or equal to p_s^H and less than B. Therefore, the revenue of S^H is $v_{[p_s^H,B]}(1,1)$ calculated by using Eq.(4) and the revenue of S^L is equally $v_{[p_s^H,B]}(1,1)$.

From (i) and (ii), when seller S^L sets start price p_s^L and seller S^H sets start price p_s^H, the expected revenue of two sellers is the same value. The expected revenue $r_{(2)}$ is expressed as

$$r_{(2)} = F_{[\underline{v},B]}(p_s^H)\ p_s^H + (1 - F_{[\underline{v},B]}(p_s^H))\ v_{[p_s^H,B]}(1,1). \tag{13}$$

a-2) Two sellers set the same start price ($p_{s1} = p_{s2}$)

Consider $r_{(0)}$, $r_{(1)}$ and $r_{(2)}$ in Eq.(10) in the case where two sellers set the same start price. The start price is called as p_s^M.

First, $r_{(0)}$ is calculated converting f and F in Eq.(9) into $f_{[\overline{v},B]}$ by using Eq.(5) and $F_{[\overline{v},B]}$ by using Eq.(6).

Secondly, consider $r_{(1)}$ in this case. Both p_s^L and p_s^H in Eqs.(11) and (12) correspond to p_s^M. The expected revenue in this case can be considered in much the same way. If a buyer has his valuation larger than or equal to p_s^M, the buyer randomly bid an auction. There are three cases where how many buyers have their valuations larger than p_s^M. Therefore, the expected revenue of the each seller, in the case where two sellers set start price p_s^M, is shown as

$$r_{(1)} = F_{[\underline{v},B]}(p_s^M)^2 \, \frac{p_s^M}{2} + 2F_{[\underline{v},B]}(p_s^M)(1 - F_{[\underline{v},B]}(p_s^M)) \; p_s^M + (1 - F_{[\underline{v},B]}(p_s^M))^2 \; v_{[p_s^M,B]}(2,2). \quad (14)$$

Finally, the expected revenue $r_{(2)}$ is calculated by substituting p_s^M to p_s^H in Eq.(13).

b) The other seller's start price is larger than the buyout price ($p_{s2} > B$)

Consider $r_{(0)}$, $r_{(1)}$ and $r_{(2)}$ in Eq.(10) in the case where start price p_{s2} of seller S_2 is larger than buyout price B. Since $p_{s2} > B$ and $p_{s1} \leq B$, the bound $p_{s1} < p_{s2}$ is satisfied. In this case, no buyers can bid the start price p_{s2} which seller S_2 sets. Therefore, only the auction of seller S_1 is bid.

$r_{(0)}$ in Eq.(10) is calculated as

$$r_{(0)} = 3(1 - F_{[\underline{v},B]}(p_s))F_{[\underline{v},B]}(p_s)^2 p_s + 3(1 - F_{[\underline{v},B]}(p_s))^2 F_{[\underline{v},B]}(p_s) v_{[p_s,B]}(2,2) + (1 - F_{[\underline{v},B]}(p_s))^3 v_{[p_s,B]}(3,2).$$

$r_{(1)}$ in Eq.(10) is calculated as

$$r_{(1)} = F_{[\underline{v},B]}(p_s)^2 p_s + 2(1 - F_{[\underline{v},B]}(p_s))F_{[\underline{v},B]}(p_s) v_{[p_s,B]}(1,1) + (1 - F_{[\underline{v},B]}(p_s))^2 v_{[p_s,B]}(2,1).$$

$r_{(2)}$ in Eq.(10) is calculated as $r_{(2)} = B$.

5.5.2 Optimum Start Price and Buyout Price

Suppose that ${p_{s2}}^*$ the optimum start price of seller S_2 is known in the case where seller S_1 sets start price p_{s1} and buyout price B. In this case, seller S_1 can calculate his expected revenue by using Eq.(10). Therefore, seller S_1 can find start price ${p_{s1}}^*$ and buyout price B^* to maximize his expected revenue.

5.6 Strategy of the Seller who does not Set a Buyout Price in the Case where the Other Seller Sets It

This section discuss the strategy of seller S_2 in the case where seller S_1 sets a buyout price in addition to a start price.

5.6.1 Expected Revenue

Seller S_2 can set the start price p_{s2} considering the start price and the buyout price which seller S_1 sets. Consider expected revenue of seller S_2 in the case where the seller S_1 sets start price p_{s1} and buyout price B.

Divide into the two cases: a) p_{s2} is less than or equal to B, b) p_{s2} is larger than B.

a) The seller's start price is less than or equal to the buyout price which the other seller sets ($p_{s2} \leq B$)

First, consider the case where start price p_{s2} which seller S_2 sets is less than buyout price B. There are four cases (i) to (iv) where how many buyers have their valuations larger than B.

(i) All three buyers have their valuations less than B

Since no buyer has his valuation larger than or equal to B, no seller can bid the auction of seller S_1 at a buyout price. Therefore, this case is regarded as the case two ascending auctions are held in stage 2. This case corresponds to the discussion in 5.5.1 a) and the revenue $r_{(0)}$ is calculated in the same way.

(ii) A buyer has his valuation larger than or equal to B

First, consider the case where a buyer who has his valuation larger than or equal to B is not risk-averse. The probability of this case is $1 - q$. This case corresponds to the discussion in 5.5.1 a) and the revenue is calculated in the same way. Therefore, the expected revenue is $r_{(1)}$ calculated using by Eqs.(11), (12) and (14).

Secondly, consider the case where a buyer who has his valuation larger than or equal to B is risk-averse. The probability of this case is q. In this case, the other two buyers participate auction of seller S_2. There are three cases where how many buyers of the two have their valuations larger than p_{s2}. The expected revenue is shown as

$$r_{s2(0)} = 2F_{[\underline{v},B]}(p_{s2})(1 - F_{[\underline{v},B]}(p_{s2}))p_{s2} + (1 - F_{[\underline{v},B]}(p_{s2}))^2 v_{[p_{s2},B]}(2, 2).$$

(iii) Two buyers have their valuations larger than or equal to B

First, consider the case where the two buyers who have their valuations larger than or equal to B are not risk-averse. The probability of this case is $(1-q)^2$. This case corresponds to the discussion in 5.5.1 a) and the revenue is calculated in the same way. The revenue in this case is $r_{(2)}$ calculated by using Eq.(13).

Secondly, consider the case where at least one buyer who has his valuation larger than or equal to B is risk-averse. The probability of this case is $1-(1-q)^2$. While risk-averse buyer purchases the good of seller S_1 at buyout price B, the other two buyers participate seller S_2's auction. Since $p_{s2} < B$, at least a buyer must has his valuation larger than p_{s2}. The expected revenue in this case is shown as

$$r_{s2(1)} = F_{[\underline{v},B]}(p_{s2})p_{s2} + (1 - F_{[\underline{v},B]}(p_{s2}))v_{[p_s^{S2},B]}(1,1).$$

(iv) All three buyers have their valuations larger than or equal to B

First, all three buyers whose valuations is larger than or equal to B are risk-neutral. The probability of this case is $(1-q)^3$. In stage 2, after the price of the auction of seller S_2 is ascending by B, a buyer purchases the good of seller S_1 at a buyout price B. The other two buyers participate the auction of seller S_2.

Secondly, consider the case where all three buyers whose valuations are larger than or equal to B are risk-averse. The probability of this case is $1-(1-q)^3$. A buyer purchases the good of seller S_1 at B.

Therefore, in both cases, the other two buyers participate the auction of seller S_2. Since $p_{s2} < B$, the two buyers have their valuations larger than p_{s2}. The expected revenue in this case is shown as

$$r_{s2(2)} = v_{[B,\overline{v}]}(2,2).$$

From (i) to (iv), the expected revenue of seller S_2 in the case where seller S_1 sets buyout price B and $p_{s2} < B$ is satisfied is calculated as

$$\begin{aligned} r_{s2} &= F(B)^3 r_{(0)} + 3F(B)^2(1-F(B))((1-q)r_{(1)} + q r_{s2(0)}) \\ &\quad +3F(B)(1-F(B))^2((1-q)^2 r_{(2)} + (1-(1-q)^2) r_{s2(1)}) \\ &\quad +(1-F(B))^3 r_{s2(2)}. \end{aligned} \tag{15}$$

b) The seller's start price is larger than a buyout price which the other seller sets ($p_{s2} > B$)

Consider the case where the start price which seller S_2 sets is larger than buyout price B which seller S_1 sets. As the discussion in 5.6 a), there are four cases (i) to (iv) where how many buyers have their valuations larger than B.

(i) All three buyers have their valuations less than B

Since $p_{s2} \geq B$, no buyer can bid the auction of S_2. Therefore, the revenue of seller S_2 is 0.

(ii) A buyer has his valuation larger than or equal to B

The auction of seller S_1 can be bought at the price of the second highest valuation of the buyers whose valuations are larger than or equal to B. Since $p_{s2} \geq B$, the other buyers cannot bid the auction of seller S_2. The expected revenue of seller S_2 is 0.

(iii) Two buyers have their valuations larger than or equal to B

A buyer of the two buyers purchase the good of seller S_1 at a buyout price B. The other two buyers participates the auction of seller S_2. A buyer of them has his valuation larger than or equal to B. Therefore, this case is divided into two cases where how many buyers of the two have their valuations larger than p_{s2}. The expected revenue in this case is shown as

$$r_{s2(1)'} = (1 - F_{[B,\overline{v}]}(p_{s2}))p_{s2}.$$

(iv) All three buyers have their valuations larger than or equal to B

A buyer of the three can buy the auction of seller S_1 at buyout price B. The other two buyers participate the auction of seller S_2. They have their valuations larger than or equal to B. Therefore, this case is divided into three cases where how many buyers of the two have their valuations larger than p_{s2}. The expected revenue in this case is shown as

$$r_{s2(2)'} = 2(1 - F_{[B,\overline{v}]}(p_{s2}))F_{[B,\overline{v}]}(p_{s2})p_{s2} + (1 - F_{[B,\overline{v}]}(p_{s2}))^2 v_{[p_{s2},\overline{v}]}(2, 2).$$

From (i) to (iv), the expected revenue of seller S_2 in $p_{s2} \geq B$ is calculated as

$$r_{s2} = 3F(B)(1 - F(B))^2 r_{s2(1)'} + (1 - F(B))^3 r_{s2(2)'}. \qquad (16)$$

5.6.2 Optimum Start Price

When start price p_{s1} and buyout price B of seller S_1 are given, seller S_2 can calculate his expected revenue by Eq.(15) or Eq.(16). Seller S_2 sets the start price ${p_{s2}}^*$ to maximize his expected revenue considering the strategy of seller S_1.

5.7 Perfect Bayesian Nash Equilibrium

This section shows the combination of sellers' strategies satisfying the perfect Bayesian Nash equilibrium.

The proposal model deals the two-stage game with incomplete information where the types of all three buyers are not given. The given information for the two sellers is the distribution function F which buyers' valuations are drawn from and the probability q that a buyer is risk-averse. In addition to it, a seller knows own valuation. They have beliefs that the given information and the expected revenues calculated by using them are always correct.

Seller S_2 selling in stage 2 knows start price p_{s1} and buyout price B which seller S_1 set in stage 1. Therefore, the optimum strategy for seller S_2 is to set start price ${p_{s2}}^*$ to maximize his revenue calculated by using Eq.(15) or Eq.(16). On the other hand, seller S_1 starting to sell from stage 1 sets start price ${p_{s1}}^*$ and buyout price B^* to maximize his revenue calculated by using Eq.(10) considering that seller S_2 always set ${p_{s2}}^*$. When ${p_{s1}}^*$ and B^* are set, the expected revenue of seller S_2 setting the start price except for ${p_{s2}}^*$ is lower than the case where he set ${p_{s2}}^*$. Therefore, seller S_2 must select ${p_{s2}}^*$ to maximize his expected revenue. Since seller S_2 sets ${p_{s2}}^*$, the expected revenue of seller S_1 setting the start price and buyout price except for combination of ${p_{s1}}^*$ and B^* is less than the case where he set ${p_{s1}}^*$ and B^*. Therefore, seller S_1 must selects the combination of ${p_{s1}}^*$ and B^* to maximize his expected revenue.

Therefore, the combination of the following combination of two strategies satisfies the perfect Bayesian Nash equilibrium: (1) seller S_1 sets start price ${p_{s1}}^*$ and buyout price B^*, (2) seller S_2 sets start price ${p_{s2}}^*$.

5.8 Evaluation of Extended Model

This section describes the experiments using the extended model.

5.8.1 Method of Experiments

The method of experiments was set as follows.

F was set to the uniform distribution function on $[100, 200]$. The smallest amount in the auction that seller can set as start price and buyout price was set to 1. The valuation of seller S_2 was set to $v_{(1)}$ in Eq.(3). Therefore, seller S_2 could set start price p_{s2} at any price on $[100, 200]$.

First, increasing start price p_{s1} of seller S_1 from $\underline{v} = 100$ to $\overline{v} = 200$, buyout price B was set to the price larger than or equal to start price p_{s1}. Secondly, buyout price B was increased from start price p_{s1} to $\overline{v}$. In these situations, the expected revenue of seller S_2 was calculated by Eq.(15) or Eq.(16). Therefore, start price ${p_{s2}}^*$ to maximize expected revenue of seller S_2 can be obtained. On the other hand, the expected revenue of seller S_1 was calculated by Eq.(10) considering start price ${p_{s2}}^*$ of seller S_2. Therefore, start price ${p_{s1}}^*$ and buyout price B^* to maximize the expected revenue of seller S_1 can be obtained.

The experiments calculate the expected revenues of sellers in the case where seller S_1 sets start price ${p_{s1}}^*$ and buyout price B^*, seller S_2 sets start price ${p_{s2}}^*$.

5.8.2 Results of Experiments

The results of the experiments are showed.

a) Seller S_1's valuation is less than $\underline{v}$ ($v_{s1} = v_{(1)}$)

Consider the strategy in the case $v_{s1} = v_{(1)}$. In this case, seller S_1 can set start price p_{s1} and buyout price B at any prices on $[100, 200]$.

Figure 12 shows the optimum start price ${p_{s1}}^*$ and buyout price B^* of seller S_1 and the optimum start price ${p_{s2}}^*$ of seller S_2 in the perfect Bayesian Nash equilibrium of each q. In all the value of q, ${p_{s1}}^* = {p_{s2}}^* = 100$. It indicates that setting the lowest start price is optimum in any cases. On the other hand, when q satisfies $q < 0.1$, buyout price B^* is $B^* > 170$. However, in the most cases, the optimum buyout price is on the interval $167 \leq B^* \leq 170$. Thus, the optimum buyout price is almost the same value.

Figure 13 shows the seller's expected revenue in the perfect Bayesian Nash equilibrium of each q. In all the value of q, the expected revenues of seller S_1

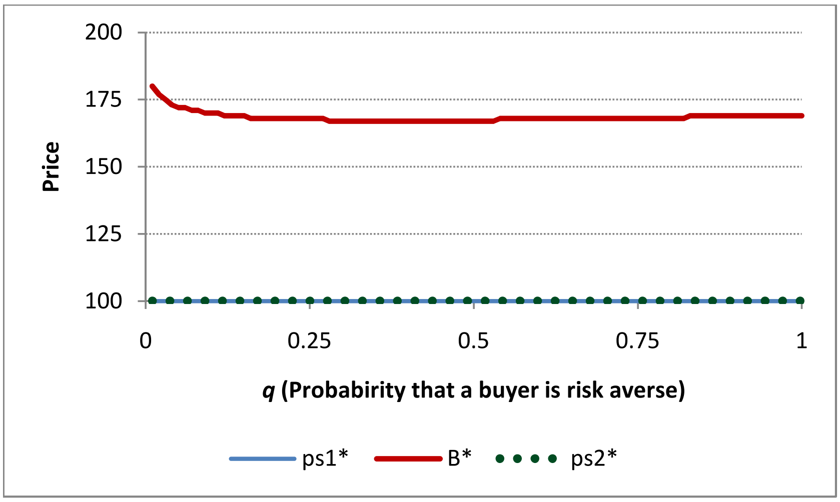

Figure 12: Optimum start price ${p_{s1}}^*$ and buyout price B^* of seller S_1 and optimum start price ${p_{s2}}^*$ of seller S_2 ($v_{s1} < 100$, $v_{s2} < 100$, F: uniform distribution on [100, 200])

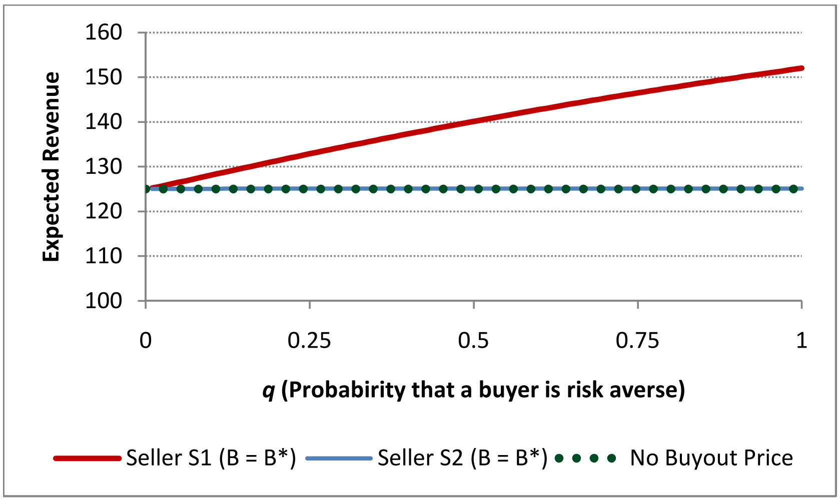

Figure 13: Comparison of seller's expected revenue ($v_{s1} < 100$, $v_{s2} < 100$, F: uniform distribution on [100, 200])

and seller S_2 in the case where seller S_1 sets a buyout price are larger than the case without a buyout price. In particular, the expected revenue of seller S_1 setting buyout price B^* increases as q increases.

b) Seller S_1's valuation is larger than or equal to $\underline{v}$ ($v_{s1} = v_{(2)}$)

Secondly, consider the optimum strategies of the perfect Bayesian Nash equilibrium in the case where $v_{s1} = v_{(2)}$. In this case, the probability q that a buyer

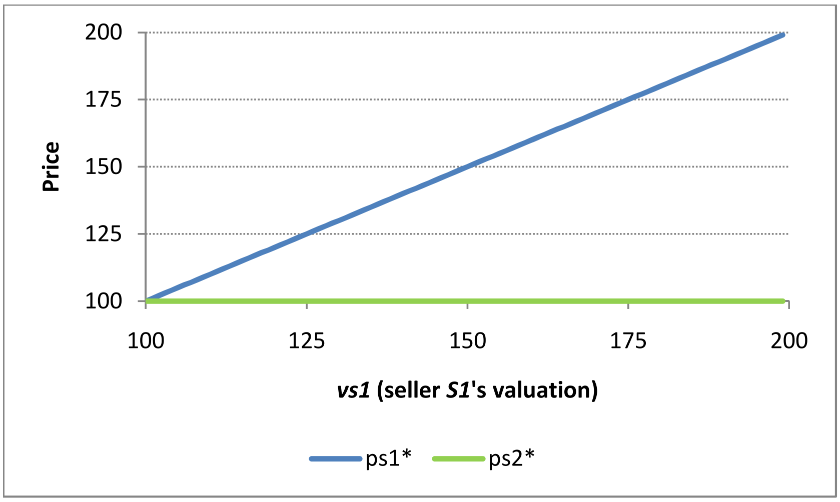

Figure 14: Optimum start price ${p_{s1}}^*$ of seller S_1 and best response start price ${p_{s2}}^*$ of seller S_2 in the case where no seller sets a buyout price ($v_{s2} < 100$, $q = 0.5$, F: uniform distribution on [100, 200])

is risk averse was set to 0.5.

Figure 14 shows the optimum strategies of the perfect Bayesian Nash equilibrium in the case without a buyout price. The optimum strategy of seller S_1 is setting the start price equal to his valuation. On the other hand, the optimum response of seller S_2 is setting the lowest start price.

Secondly, the experiment in the case where seller S_1 sets a buyout price was conducted.

Figure 15 shows the optimum start price ${p_{s1}}^*$ and buyout price B^* of seller S_1 and the optimum start price ${p_{s2}}^*$ of seller S_2 in the perfect Bayesian Nash equilibrium. Start price ${p_{s1}}^*$ is always equal to the valuation of seller S_1. On the other hand, optimum buyout price B^* is 167 when the valuation of seller S_1 v_{s1} is from 100 to 118. When the valuation satisfies $v_{s1} \leq 150$, ${p_{s1}}^* = v_{s1}$ and $B^* > {p_{s1}}^*$. On the other hand, when $v_{s1} \geq 150$, $B^* = {p_{s1}}^* = v_{s1}$. In other words, optimum strategy for seller S_1 who has his valuation $v_{s1} \geq 150$ is the fixed-price selling at a buyout price equal to his valuation. To the strategy of seller S_1, the optimum strategy of seller S_2 is setting start price 100 which is the lowest value in F.

Figure 16 shows the difference between B^* and ${p_{s1}}^*$ set by seller S_1. The

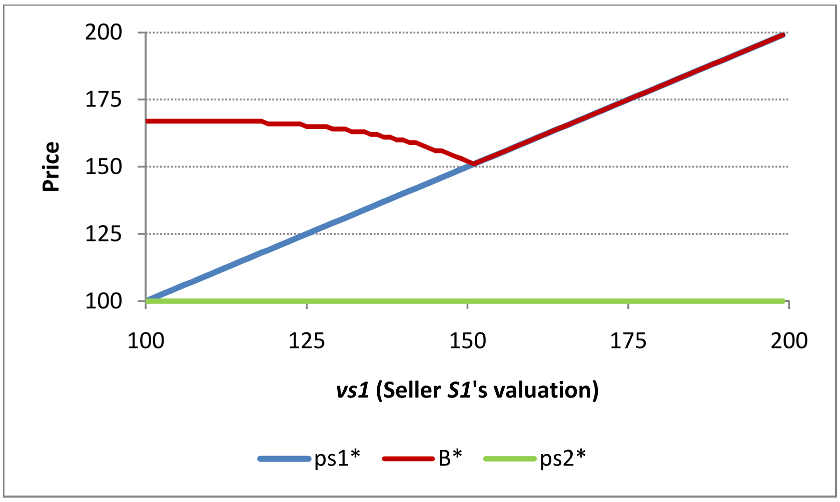

Figure 15: Optimum start price ${p_{s1}}^*$ and buyout price B^* of seller S_1 and optimum start price ${p_{s2}}^*$ of seller S_2 ($v_{s2} < 100$, $q = 0.5$, F: uniform distribution on [100,200])

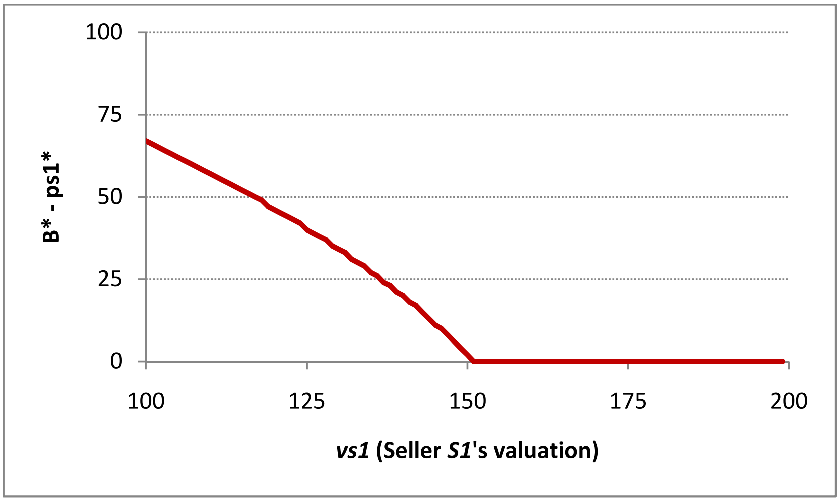

Figure 16: Difference between B^* and ${p_{s1}}^*$ on each valuation of seller S_1 ($v_{s2} < 100$, $q = 0.5$, F: uniform distribution on [100, 200])

difference decreases as the seller's valuation increases. When the seller has his valuation larger than the threshold value, fixed-price selling as $B^* - {p_{s1}}^* = 0$ is the optimum strategy.

Figure 17 shows the comparison of seller S_1's revenue. The figure indicates that, by setting the buyout prices, the revenue of seller S_1 can be improved comparing the case without a buyout price.

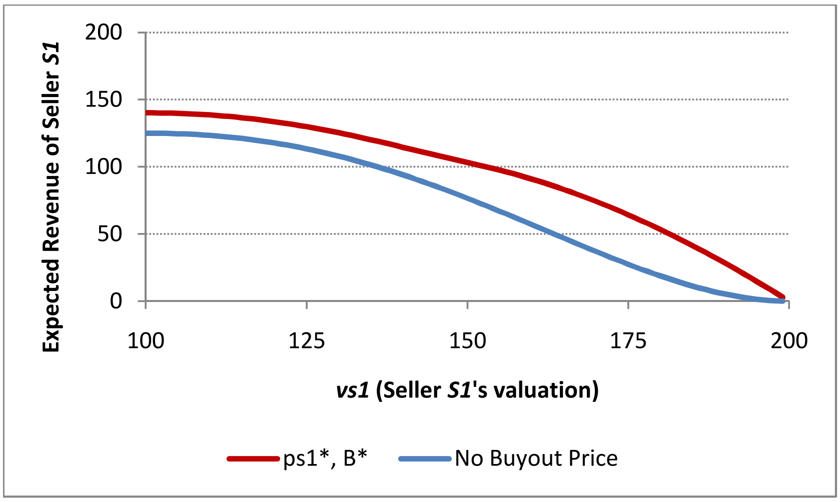

Figure 17: Comparison of seller S_1's revenue ($q = 0.5$, F: uniform distribution on [100, 200])

Figure 18 shows the increase of seller S_1's expected revenue by setting the optimum buyout price. The axis of ordinate in the figure shows "the expected revenue in the case where the seller sets the buyout price" - "the expected revenue in the case where no seller sets a buyout price". The interesting results are that a seller who has a large valuation to his good can improve his revenue the most. If the seller sells the good without a buyout price, the optimum strategy is to set a start price equal to his valuation. On the other hand, if the seller sells the good with a buyout price, the optimum strategy is to set both buyout price and start price equal to his valuation. A high start price does not appeal to buyers, but a high buyout price appeals to the buyers who are risk-averse and have their valuation larger than or equal to the buyout price. Therefore, the seller can improve his revenue by setting the buyout price.

Figure 19 shows the comparison of expected total revenue. The figure shows that setting the buyout price and the start prices in the perfect Bayesian Nash equilibrium improves the total revenue. Compared to the case without buyout price, the optimum price setting of start price does not differ. Therefore, setting buyout price in addition to the start prices improves the total revenue.

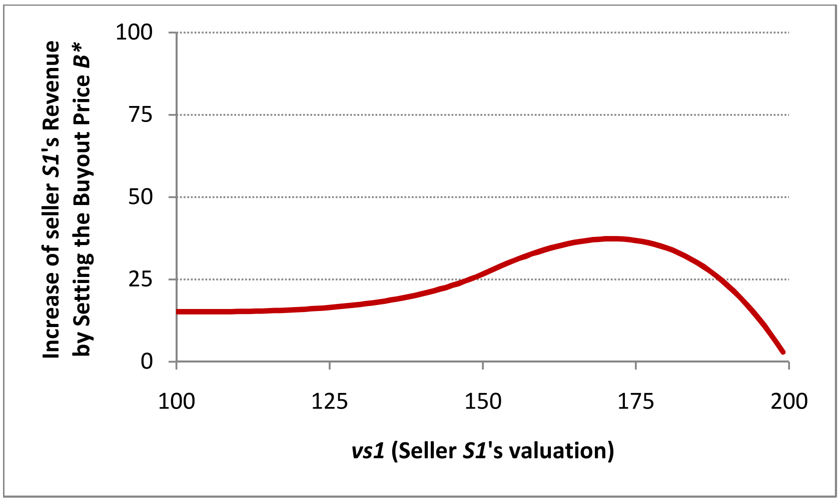

Figure 18: Increase of seller S_1's expected revenue by setting the optimum buyout price ($v_{s2} < 100$, $q = 0.5$,F: uniform distribution on [100, 200])

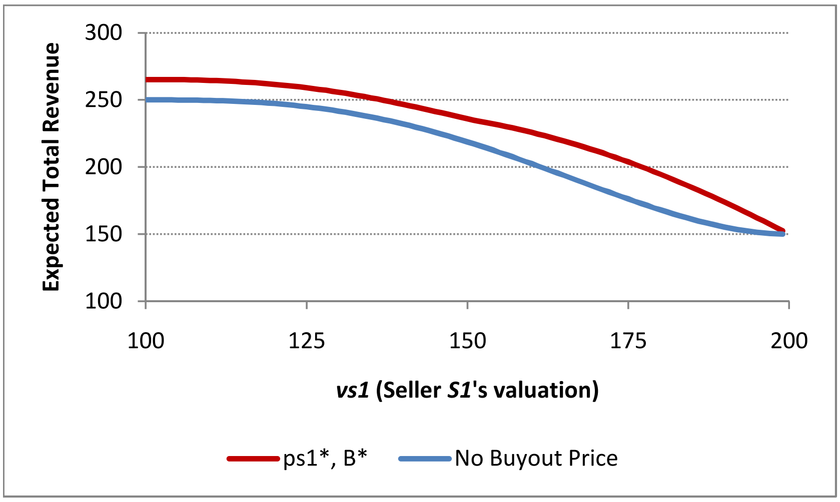

Figure 19: Comparison of expected total revenue (F: uniform distribution on [100, 200], $v_{s2} < 100$, $q = 0.5$)

5.8.3 Consideration

The result of the experiments of the extended model indicates the following things.

When sellers set the start prices and the buyout price in the perfect Bayesian Nash equilibrium, the total revenue of sellers is improved by setting a buyout price. The revenue of seller S_1 increases as q increases. In particular, the seller S_1 who has his large valuation can increase the most by fixed-price selling at the buyout price. On the other hand, the revenue of seller S_2 is slightly improved.

From the result of data analysis, many sellers who select fixed-price selling by setting buyout price equal to start price are observed. In the analysis of the model, when the valuation of the seller is larger than the threshold value, setting start price and buyout price equal to his valuation is optimum strategy. Compared to the case where the seller sets start price equal to his valuation and does not set a buyout price, the expected revenue of the seller is improved.

Chapter 6 Discussion

This chapter discusses the result of data analysis in Chapter 3 and the result of experiments using the extended model in Chapter 5.

6.1 Comparison between Data and Model

First, the author does a comparison of the result of analysis between Data and Model. The actual data tells that the buyout option is used in 55 % of auctions, while the buyout option is not used in 45% of auctions. In the actual data of auctions without buyout prices, many sellers set a quite low start price (TYPE 1). On the other hand, in the actual data of auctions with buyout prices, many sellers set a buyout price almost equal to the start prices: the fixed-price selling (TYPE 2).

These results are compared to the result of analysis using the proposal model as follows. The result of the experiments using the extended model indicates that it is the optimum strategy for the first seller who has his large valuation to set a buyout price equal to the start prices: the fixed-price selling. This result corresponds to the TYPE 2 obtained from the actual data. On the other hand, for the second seller who does not set a buyout price, the result indicates that it is the optimum strategy to set the lowest start price. This result corresponds to the TYPE 1 obtained from the actual data.

6.2 Optimum Price Setting

This section describes the optimum price setting indicated by the proposal model.

When the first seller sets a buyout price, setting the lowest start price is the best response for the second seller. The first seller sets the combination of start price and buyout price to maximize his expected revenue considering the best response of the second seller.

The fixed-price selling at the buyout price is valid for the first seller who has his large valuation. On the other hand, for the first seller who does not have his large valuation, the optimum price setting is to set a start price at the price equal to his valuation and a buyout price at the price larger than it.

The combination of the strategies satisfies the perfect Bayesian Nash equilibrium. If sellers select the strategies, the total revenue of the sellers is higher than the case no seller sets a buyout price.

Chapter 7 Conclusion

It has been reported that the trades having buyout options are increasing in Internet auctions. Understanding how a buyout option is used and how it affects the seller's revenue is to design the future auction markets. The author has analyzed the actual auction data, characterized the typical strategies of the seller, built models, and carried out experiments on them, which leads to deeper understanding of the market where ascending auction and fixed-price selling simultaneously exist.

The contributions of this research are summarized as follows.

Presenting major strategies of sellers in an Internet auction market 11,921 auction data obtained from an actual Internet auction site were examined by focusing on the setting of start price and buyout price. The results of data analysis show the two major strategies of the sellers in the market as follows: (1) many of sellers who set buyout prices sell by fixed-price selling at a buyout price, (2) many of sellers who do not set buyout prices set start prices at quite low price.

Proposing the model to explain the coexistence of two type sellers The author can successfully provide a model able to explain the situation where the both types of sellers using the buyout option and not using the buyout option simultaneously exist. The model supposes a two-stage game where two sellers arrive sequentially. First, the case where the seller's strategy is restricted to the two strategies obtained from the actual data was discussed. In this case, if the probability that a buyer is risk-averse is quite high, both two sellers can benefit by selling a good by using a buyout option and selling another good by using an ascending auction. Secondly, the strategies in the perfect Bayesian Nash equilibrium are showed. If the first seller has a large valuation to his good, the combination of the strategies in the equilibrium corresponds to the two major strategies in the actual data. When sellers select strategies satisfying the perfect Bayesian Nash equilibrium, the total revenue of the sellers is higher than the case no seller sets a buyout price.

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